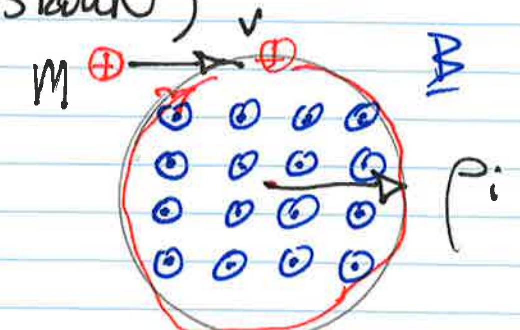
# GPAP SUMMER SCHOOL 2021

## #1 FUNDAMENTALS of PLASMAS (M. BROWN)

OVERVIEW:
- SINGLE PROTON ✓
- PLASMA FREQUENCY ✓
- DEBYE LENGTH ✓
- COLLISIONS ✓
- CONNECTION




### (1) SINGLE PROTON

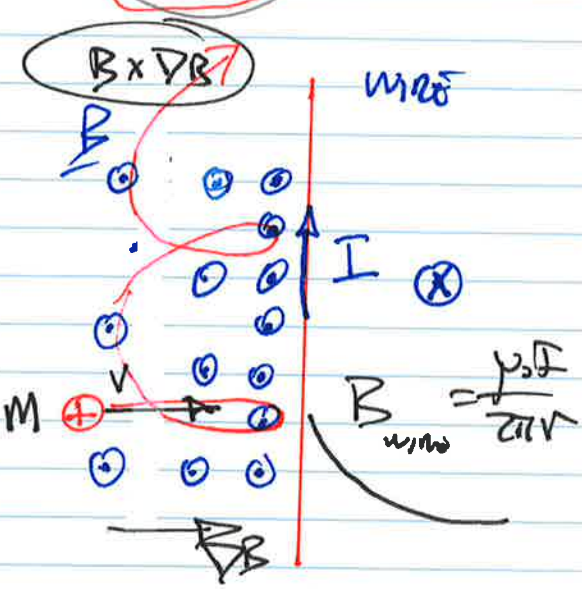
$\underline{F} = q(\underline{E} + \underline{v} \times \underline{B})$ LORENTZ

$F = q(\underline{v} \times \underline{B}) = qvB = ma = m\frac{v^2}{r}$

$$\boxed{r = \frac{mv}{qB}} \checkmark$$

$V = \omega r = \frac{2\pi}{T} r \rightarrow \boxed{\omega = \frac{qB}{m}}$

IONS → "LIONS"




DRIFTS: LIBBY TOLMAN ✓

### (2) PLASMA FREQUENCY (COLLECTIVE)

DENSITY $n = \frac{\#}{VOL}$

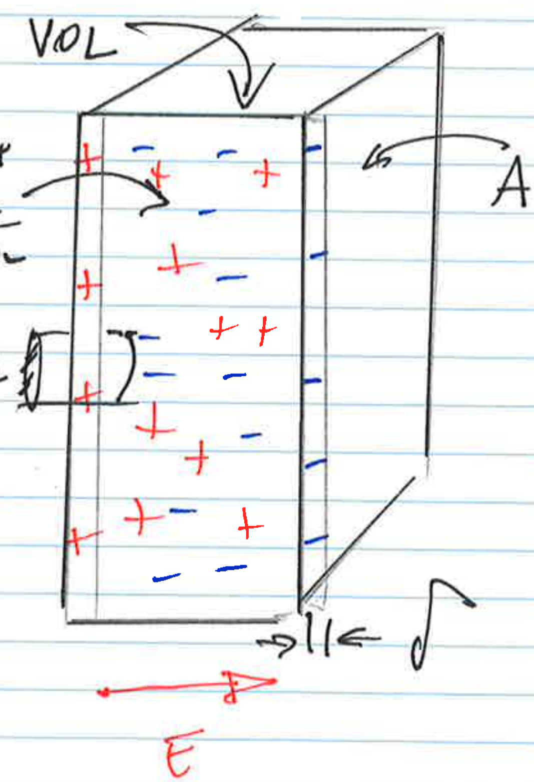
$E = \frac{\sigma}{\epsilon_0}$ $\quad \oint E \cdot da = \frac{Q_{in}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$ → a

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad \overset{\shortparallel}{Ea}$

$= \frac{ne\delta}{\epsilon_0}$

$M_{e^-}^{TOT} \ddot{a} = \underbrace{m_e n (VOL)}_{M_{TOT}} \ddot{\delta} = F = QE$

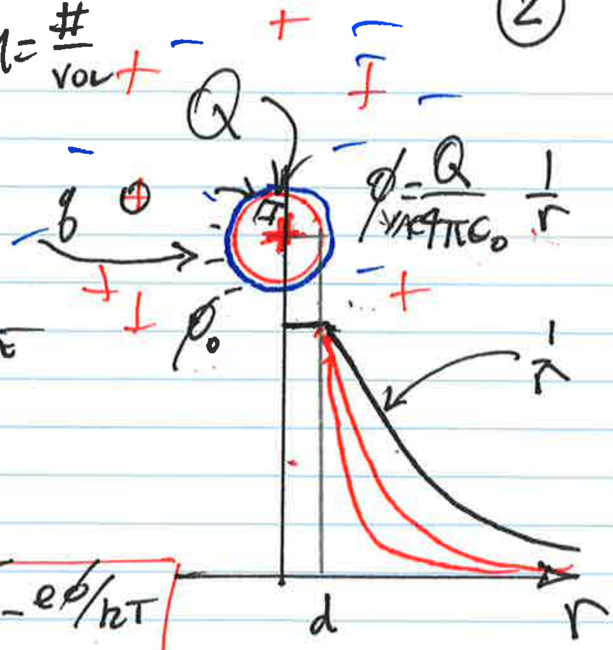
RESTORING $= -\underbrace{ne(VOL)}_{Q} \frac{ne\delta}{\epsilon_0}$




so $\boxed{\ddot{\delta} = \frac{-ne^2}{m_e\epsilon_0}\delta}$ $\therefore \boxed{\omega_{pe}^2 = \frac{ne^2}{m_e\epsilon_0}}$

$n = \frac{\#}{\text{VOL}}$

## (3) DEBYE SHIELDING (FLUID)




$$m\frac{dv}{dt} = qE - \frac{\nabla p}{n} = 0$$

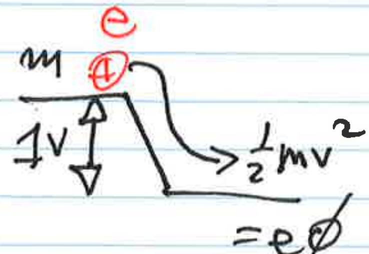
STEADY STATE

$$\text{so} \quad qE = \frac{\nabla p}{n} \qquad E = -\nabla\phi$$

$$-nq\nabla\phi = kT\nabla n \quad \text{ISOTHERMAL}$$

$$\nabla = \frac{\partial}{\partial r}\ldots \qquad \frac{dn}{n} = \frac{-q\,d\phi}{kT} \rightarrow \boxed{n = n_0 e^{-e\phi/kT}}$$

BOLTZMANN




$kT = e\phi \rightarrow 11{,}600°K$ (1 eV)

☆ APPROXIMATION, PLASMA $e\phi \ll kT$.

$$n \cong n_0\left(1 - e\phi/kT\right)$$

WHAT IS $\phi(r)$? $\quad \nabla\cdot E = \frac{\rho}{\epsilon_0}$ & $E = -\nabla\phi$

POISSON EQ. $\nabla^2\phi = \frac{-q}{\epsilon_0}(n_0 - n_e)$ (IONS, ELECTRONS)

☆ QUASI-NEUTRAL $q_i n_i + q_e n_e \cong 0$

$$\text{so} \quad \nabla^2\phi \cong -\frac{n_0 e}{\epsilon_0}\left(1 - \left(1 + e\phi/kT\right)\right) = \frac{-n_0 e^2\phi}{kT\epsilon_0} \sim \frac{\phi}{\ell^2}$$

$$\nabla^2\phi = \frac{\phi}{\lambda_D^2} \qquad \phi(r) = \frac{\rho_0}{r}e^{-r/\lambda_D} \qquad \boxed{\lambda_D^2 = \frac{kT\epsilon_0}{n_0 e^2}}$$

SPHERICAL COORDS

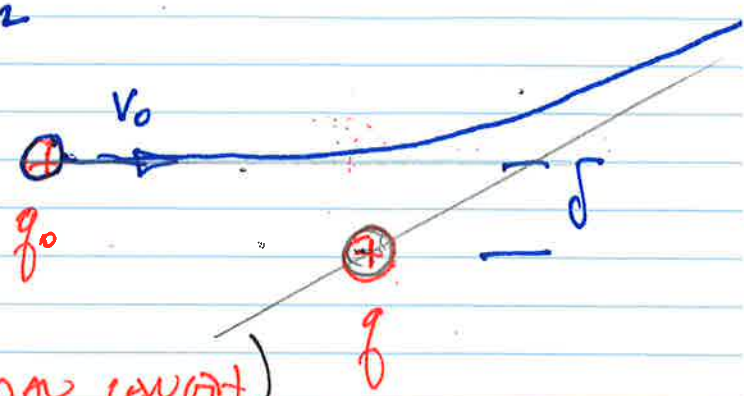
$$= \boxed{\frac{Q}{4\pi\epsilon_0 r}e^{-r/\lambda_D}}$$

$$\boxed{\Lambda = n\lambda_D^3} \Rightarrow \text{DIM-LESS}$$

\# of ELECTRONS IN A DEBYE CUBE.

## (4) COLLISIONS

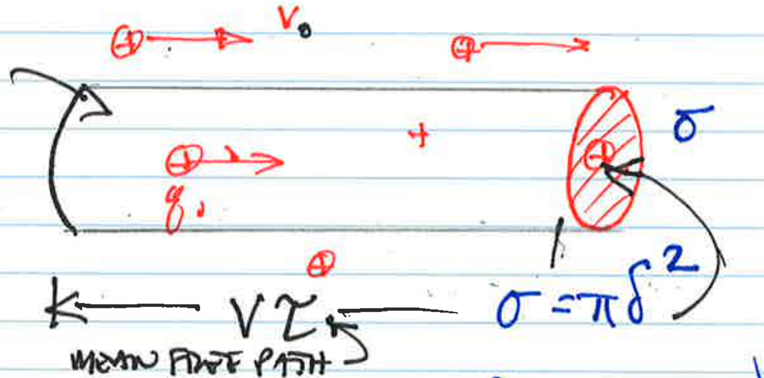
$KE = \frac{1}{2} m v^2$




$\frac{1}{2} m v_0^2 \cong \frac{q q_0}{4\pi\epsilon_0 \delta}$ COULOMB.

$\delta = 2 q q_0 / 4\pi\epsilon_0 m v_0^2$ (LANDAU LENGTH)

$v_{th} = \sqrt{\frac{kT}{m}}$




$n \sigma v_0 \tau = 1$ ($\sigma$: AREA; $v_0\tau$: length; VOL)

$\tau$ = COLLISION TIME

COLLISION FREQ $= \nu = \frac{1}{\tau}$

$\sigma = \pi\delta^2 = \pi \, 4 q^2 q_0^2 / (4\pi\epsilon_0)^2 m^2 v_0^4$

$q = q_0$

$$\nu_c = n \sigma v_0$$
$$= \pi \, n 4 e^4 / (4\pi\epsilon_0)^2 m^2 v_0^3$$
$$= \boxed{n e^4 / m^2 v_0^3 (4\pi\epsilon_0^2)}$$

$\nu_c \sim 1 / T^{3/2}$

OFF by $\boxed{2 \ln\Lambda} \cong 20$; $v_0^3 \rightarrow \left(\frac{kT}{m}\right)^{3/2}$

## (5) CONNECTION

$$\frac{\omega_{pe}}{\nu_{col}} = \left(\frac{n e^2}{m\epsilon_0}\right)^{1/2} \left(\frac{m^2 v_0^3 (4\pi\epsilon_0^2)}{e^4 n}\right) = \frac{4\pi\epsilon_0^{3/2} m^{3/2} v_0^3}{n^{1/2} e^3}$$

NOTE: $\Lambda \doteq n\lambda_D^3 = n \left(\frac{\epsilon_0 kT}{n e^2}\right)^{3/2} \Rightarrow \boxed{4\pi\Lambda = \frac{\omega_{pe}}{\nu_{col}}} \gg 1$

DEF. of PLASMA: ASSEMBLAGE of CHARGED PARTICLES EXHIBITING COLLECTIVE BEHAVIOR. $e\phi \ll kT$. $\Lambda \gg 1$

m = electron
M = proton

# Fundamental Plasma Scales

| | Frequencies: $f_{coll} < \omega_{cyc} < \omega_{plasma}$ | | | Lengths: $\lambda_D < \rho_{cyc} < \delta_{inertia}$ | | | Velocities: $V_{th} < V_{Alf} < c$ | | |
|---|---|---|---|---|---|---|---|---|---|
| Electrons | $\sim \frac{1}{T^{3/2}}$ | $\frac{qB}{m}$ | $\sqrt{\frac{ne^2}{m\epsilon_0}}$ | $\sqrt{\frac{kT\epsilon_0}{ne^2}}$ | $\frac{mv}{qB}$ | $c/\omega_{pe}$ | $\sqrt{\frac{kT_e}{m}}$ | | $3\times10^8 \frac{m}{s}$ |
| Proton | | $\frac{qB}{M}$ | $\sqrt{\frac{ne^2}{M\epsilon_0}}$ | | $\frac{Mv}{qB}$ | $c/\omega_{pi}$ | $\sqrt{\frac{kT_i}{M}}$ | $\frac{B}{\sqrt{\mu M n_0}}$ | |

$f_{ci} \cong 1500\ \text{Hz/Gauss}$

$f_{pe} \cong 9000 \sqrt{n_{cm}}\ \text{Hz}$

$\rho_{ci} \cong 100\ \text{cm} \frac{\sqrt{T_{eV}}}{B_G}$

$V_{th,p} \cong 10 \frac{km}{s} \sqrt{T_{eV}}$

PLASMA: NEED $\frac{\omega_{pe}}{f_{coll}} \gg 1$, $\Lambda = n\lambda_D^3 \gg 1$, $kT \gg e\phi$

VELOCITY = FREQ × LENGTH eg $c = f\lambda$

NOTE: $V_e = \omega_{pe}\lambda_D$ and $V_e = \omega_{ce}\rho_e$ so $\frac{\omega_{pe}}{\omega_{ce}} = \frac{\rho_e}{\lambda_D}$

$V_A = \omega_{ci}\delta_i$ and $V_i = \omega_{ci}\rho_i$ so $\frac{\rho_i}{\delta_i} = \frac{V_i}{V_A} = \sqrt{\beta}$