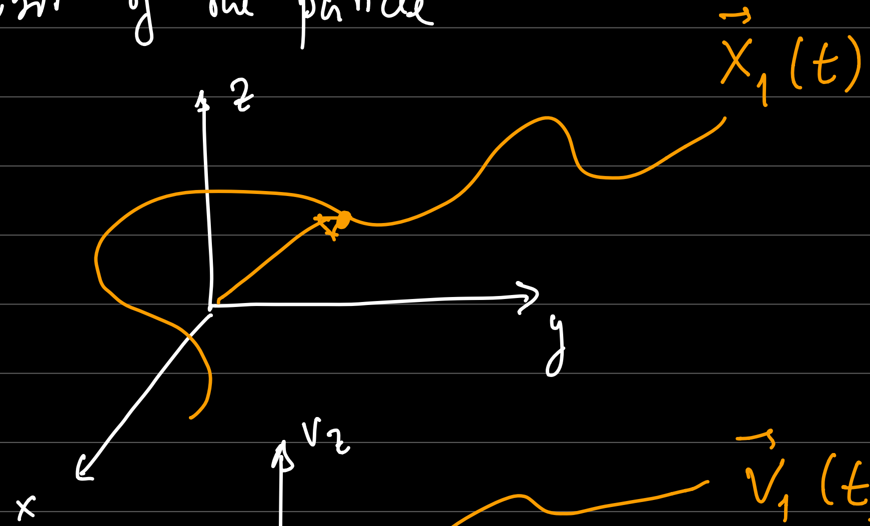
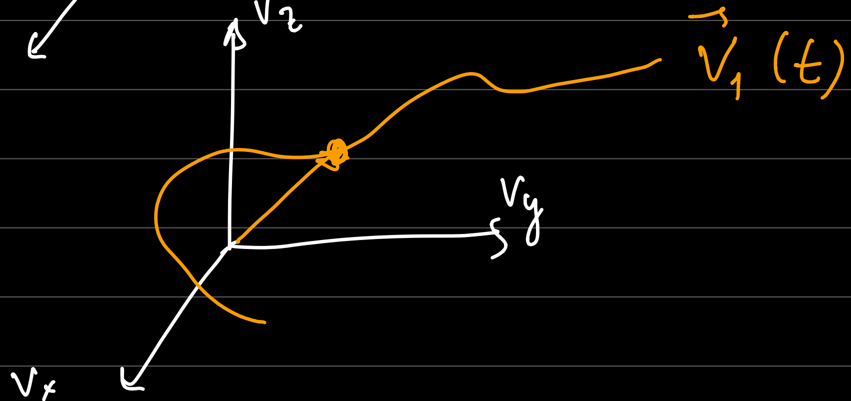
Orbit of one particle





$$N(\vec{x}, \vec{v}, t) = \delta[\vec{x} - \vec{X}_1(t)]\, \delta[\vec{v} - \vec{V}_1(t)]$$

Density of this one particle in phase-space

$$\int N(\vec{x}, \vec{v}, t)\, d^3\vec{x}\, d^3\vec{v} = 1$$

$$N_s(\vec{x}, \vec{v}, t) = \sum_{i=1}^{N_0} \delta[\vec{x} - X_i(t)]\, \delta[\vec{v} - \vec{V}_i(t)]$$

$$N(\vec{x}, \vec{v}, t) = \sum N_s(\vec{x}, \vec{v}, t)$$

$s = e, i$

$$\begin{cases} \dfrac{d\vec{X}_i}{dt} = \vec{V}_i(t) & \text{for particle } i \\[2ex] m_s \dfrac{d\vec{v}_i}{dt} = q_s E^{m}[X_i(t), t] + \dfrac{q_s}{c}\vec{V}_i(t) \times \vec{B}^{m}[\vec{X}_i(t), t] \end{cases}$$

microscopic

$$\nabla \cdot \vec{E}^m = 4\pi \rho^m(\vec{x}, t) \quad ; \quad \nabla \cdot \vec{B}^{(m)} = 0$$

$$\nabla \times \vec{E}^m = -\frac{1}{c}\frac{\partial \vec{B}^m(\vec{x}, t)}{\partial t} \quad ; \quad \nabla \times \vec{B}^m = \frac{4\pi}{c}\vec{J}^m + \frac{1}{c}\frac{\partial}{\partial t}\vec{E}^m(\vec{x}, t)$$

$$\rho^m(\vec{x}, t) = \sum_{e,i} q_s \int N_s(\vec{x}, \vec{v}, t)\, d\vec{v}$$ charge density

$$J^m(\vec{x}, t) = \sum_{e,i} q_s \int \vec{v}\, N_s(\vec{x}, \vec{v}, t)\, d\vec{v}$$ microscopic current density

$$N_s(\vec{x}, \vec{v}, t) = \sum_{i=1}^{N_0} \delta[\vec{x} - \vec{X}_i(t)]\, \delta[\vec{v} - \vec{V}_i(t)]$$

$$\frac{\partial N_s(\vec{x}, \vec{v}, t)}{\partial t} = -\sum_{i=1}^{N_0} \frac{d\vec{X}_i}{dt} \cdot \nabla_x \delta[\vec{x} - \vec{X}_i(t)]\, \delta[\vec{v} - \vec{V}_i(t)]$$
$$- \sum_{i=1}^{N_0} \frac{d\vec{V}_i}{dt} \cdot \nabla_v \delta[\vec{x} - \vec{X}_i(t)]\, \delta[\vec{v} - \vec{V}_i(t)]$$

$$a\,\delta(a-b) = b\,\delta(a-b)$$

$$\frac{\partial N_s}{\partial t}(\vec{x},\vec{v},t) = -\vec{v}\cdot\nabla_x \sum_{i=1}^{N_0} \delta[\vec{x}-\vec{X}_i(t)]\,\delta[\vec{v}-\vec{V}_i(t)]$$

$$-\left[\frac{q_s}{m_s}\vec{E}^m(\vec{x},t) + \frac{q_s}{m_s c}\vec{v}\times\vec{B}^m(\vec{x},t)\right]\cdot\nabla_v \sum_{i=1}^{N_0} \delta[\ldots]\delta[\ldots]$$

$$\boxed{\frac{\partial N_s^{(\vec{x},\vec{v},t)}}{\partial t} + \vec{v}\cdot\nabla_x N_s + \frac{q_s}{m_s}\left(\vec{E}^m + \vec{v}\times\vec{B}^m\right)\cdot\nabla_v N_s = 0}$$

Klimontovich eq.

$\Delta\vec{x}\,\Delta\vec{v}$

$$f_s(\vec{x},\vec{v},t) \equiv \langle N_s(\vec{x},\vec{v},t)\rangle$$

↳ distribution function: the # of particles of species s per unit volume in both config. space and vel. space.

$$N_s(\vec{x},\vec{v},t) = f_s(\vec{x},\vec{v},t) + \delta N_s(\vec{x},\vec{v},t)$$

$$\langle \delta N_s \rangle = 0$$

$$\vec{E}^m = \vec{E}(\vec{x}, \vec{v}, t) + \delta \vec{E}(\vec{x}, \vec{v}, t)$$
$$\vec{B}^m = \vec{B}(\vec{x}, \vec{v}, t) + \delta \vec{B}(\vec{x}, \vec{v}, t)$$

$$\vec{E} = \langle \vec{E}^m \rangle \quad ; \quad \vec{B} = \langle \vec{B}^m \rangle$$
$$\langle \delta \vec{E} \rangle = 0 \quad ; \quad \langle \delta \vec{B} \rangle = 0$$

$$\left\langle \frac{\partial N_s}{\partial t} \right\rangle = \frac{\partial f_s}{\partial t}$$

$$\frac{\partial f_s}{\partial t} + \vec{v} \cdot \nabla_x f_s + \frac{q_s}{m_s}\left(\vec{E} + \frac{\vec{v} \times \vec{B}}{c}\right) \cdot \nabla_v f_s = -\frac{q_s}{m_s}\left\langle \left(\delta \vec{E} + \frac{\vec{v} \times \delta \vec{B}}{c}\right) \cdot \nabla_v \delta N_s \right\rangle$$

Plasma kinetic equation $\qquad$ C_s collision op.

smooth: response to collective effects

spikey: discrete particle interactions.

$$\int \frac{\partial f_s}{\partial t}(\vec{x}, \vec{v}, t) + \vec{v} \cdot \nabla_x f_s + \frac{q_s}{m_s}\left(\vec{E} + \frac{\vec{v} \times \vec{B}}{c}\right) \cdot \nabla_v f_s = 0$$

Vlasov equation.

$$n_s \equiv \int f_s \, d^3\vec{v}$$

$$\frac{\partial n_s}{\partial t} + \nabla \cdot (\vec{u} n_s) = 0$$

fluid velocity

$\boxed{\text{Nicholson}}$ → reference. + Prof. Kunz's notes.