Kinetic Plasma Turbulence in Space and Astrophysical Plasmas

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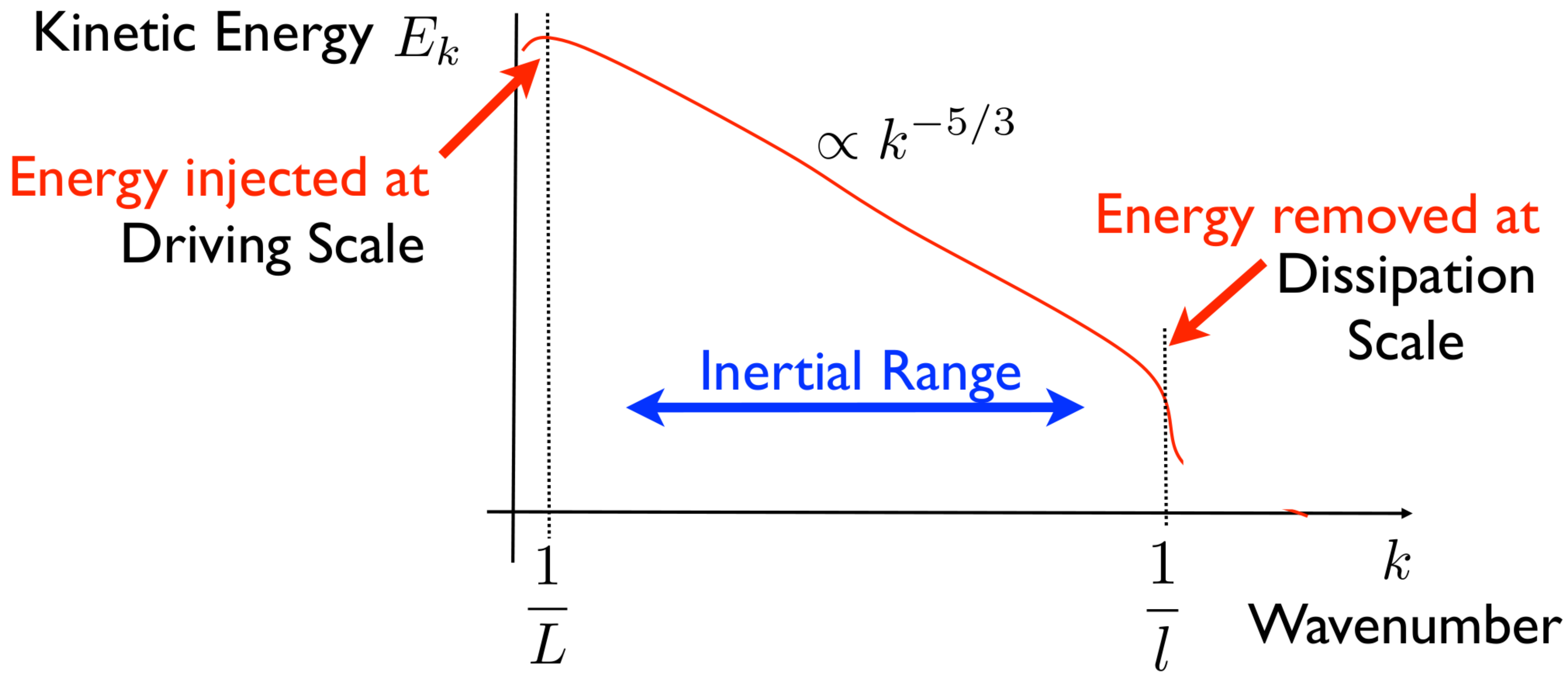
Outline

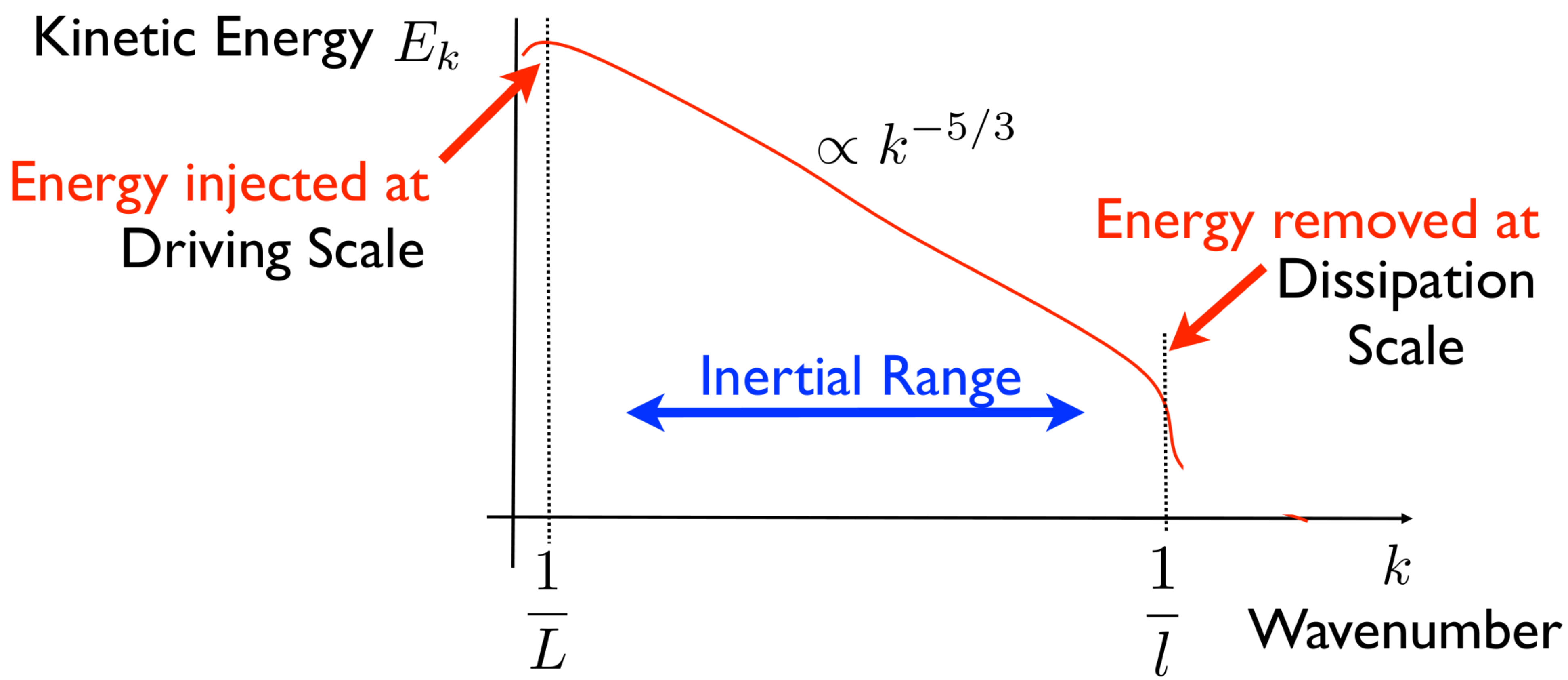
- What happens in weakly collisional plasmas?
- What happens when the cascade reaches kinetic scales?
- What about other dissipation mechanisms?
- How do we diagnose kinetic dissipation?
- What about instabilities?

What happens in weakly collisional plasmas?

Energy Spectrum

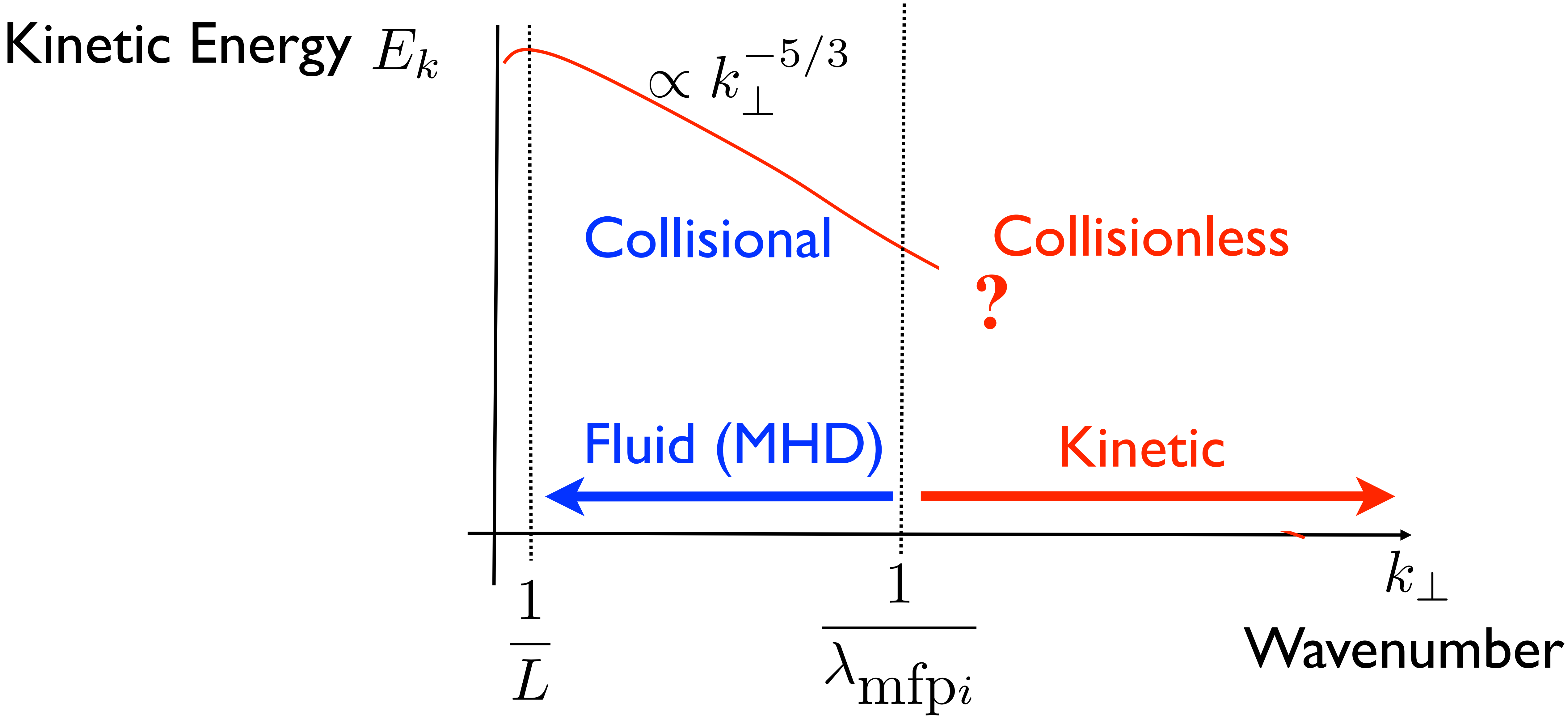


MHD energy spectrum



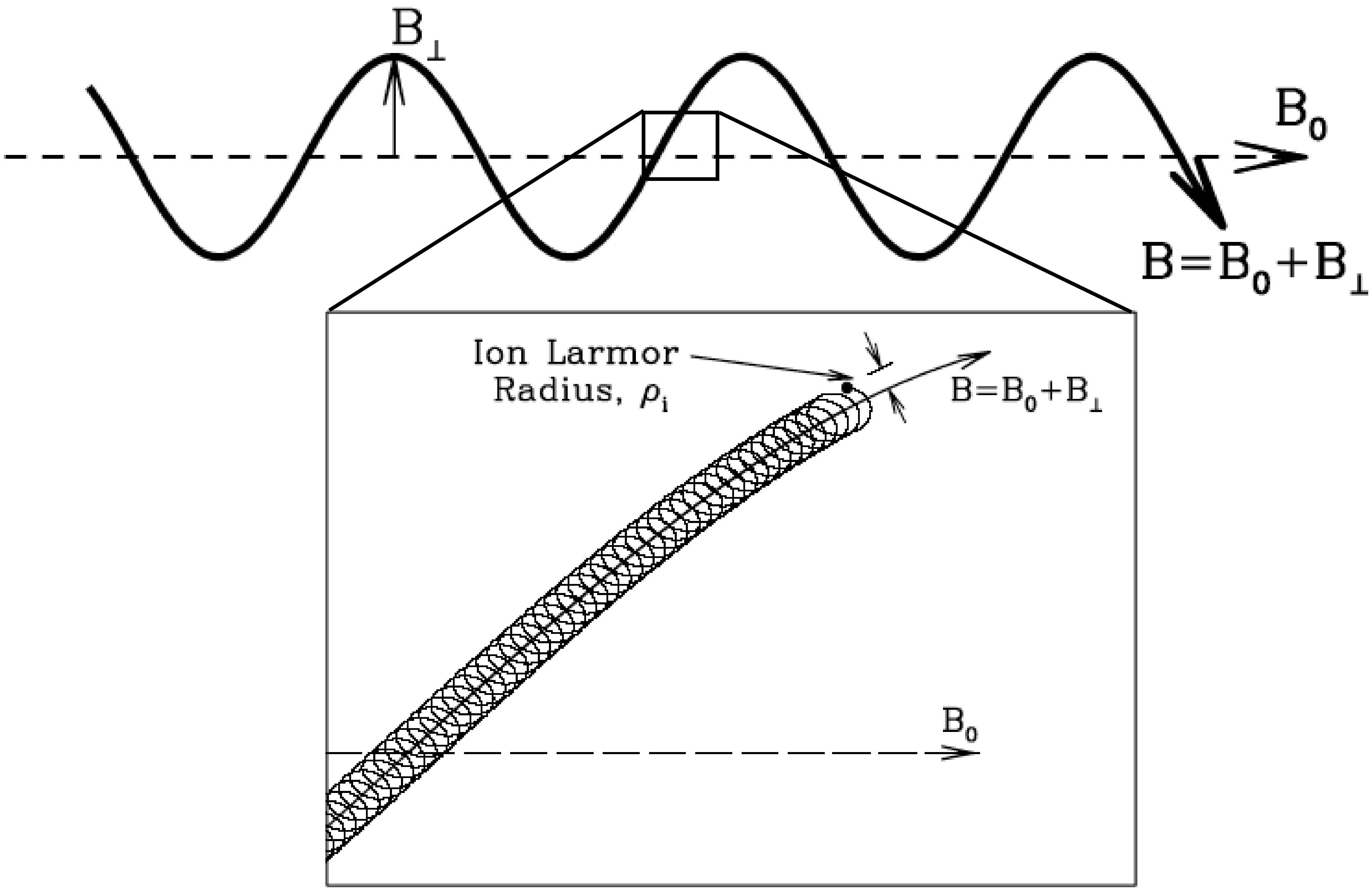
What sets the dissipation scale?

Beginnings of kinetic turbulence

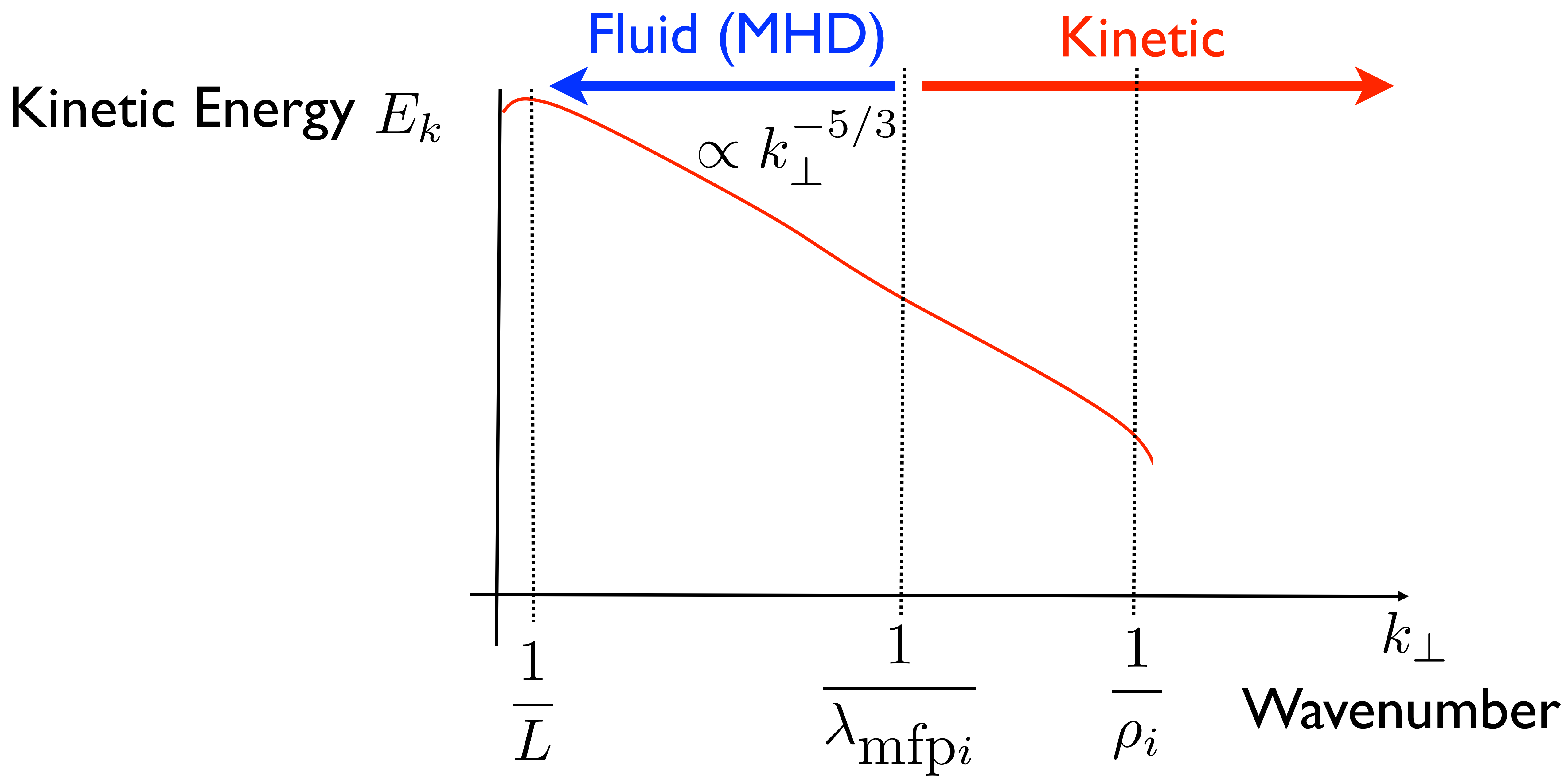


Importance of the gyroradius

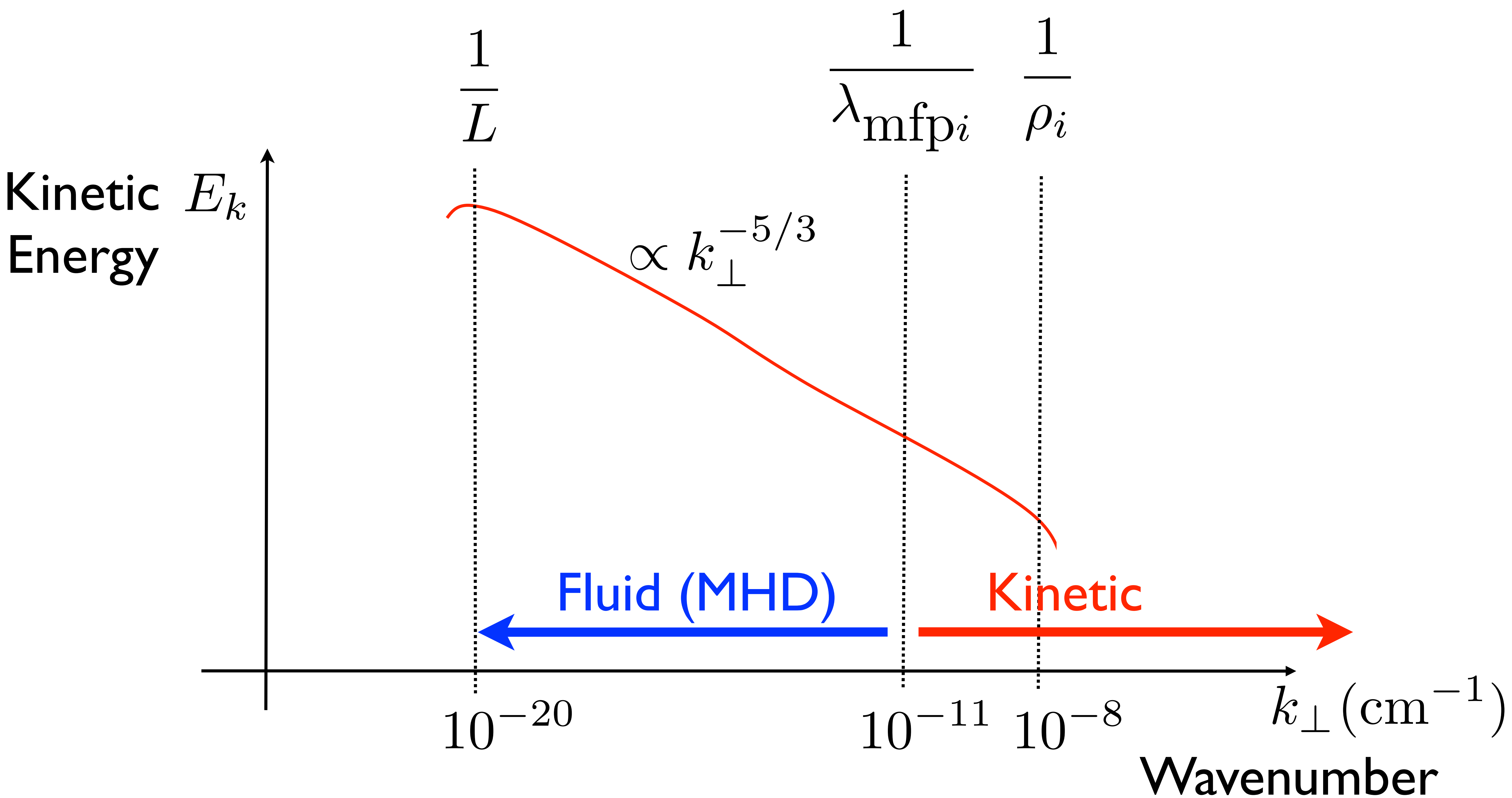
At collisionless scales, $l < \lambda_{mfp}$ the Alfven wave cascade continues undamped to the scale of the ion gyroradius, $\rho_i = v_{thi}/\Omega_{ci}$ [Schekochihin et al (2009)]



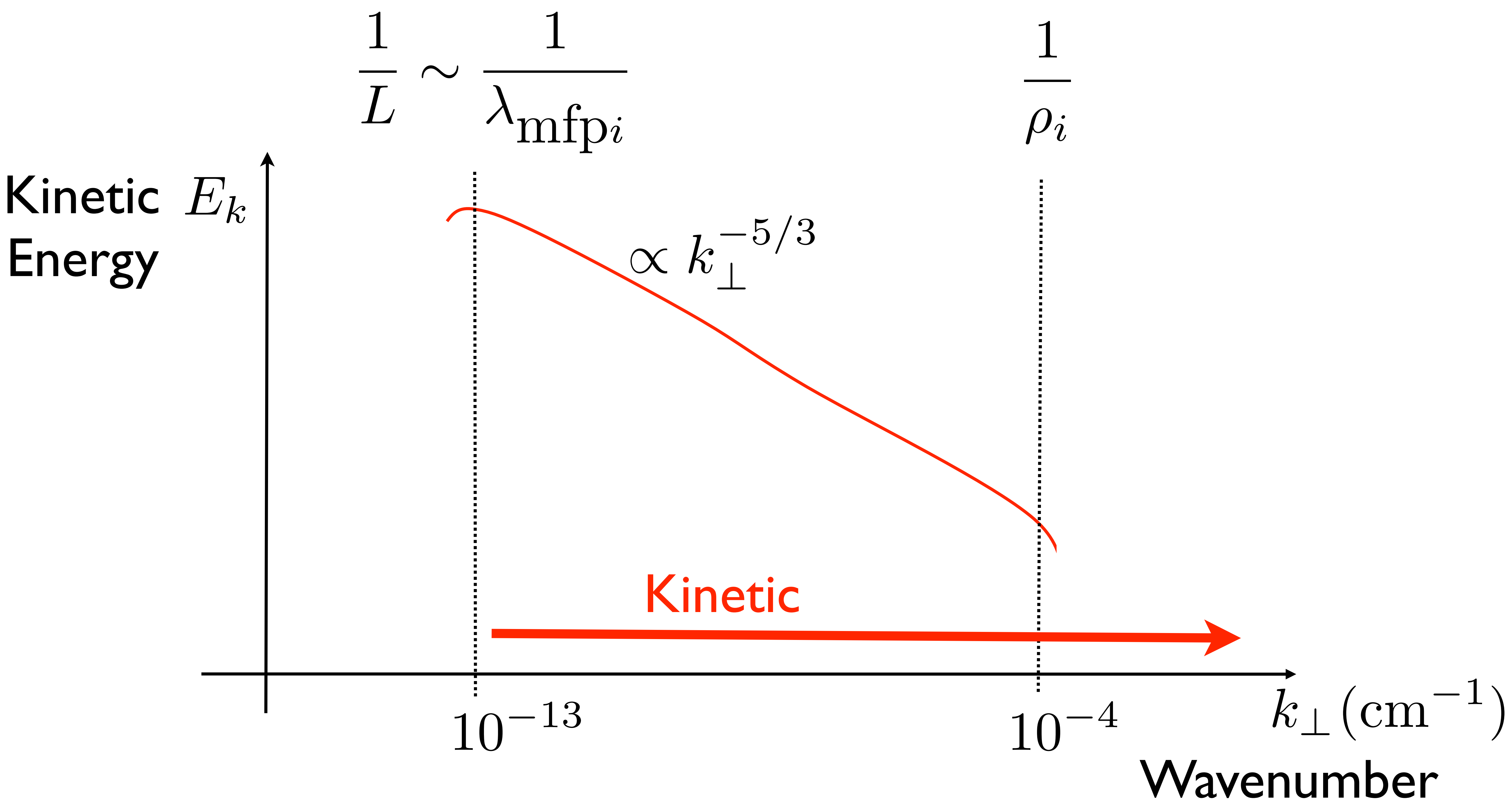
Cascade to the ion gyroradius



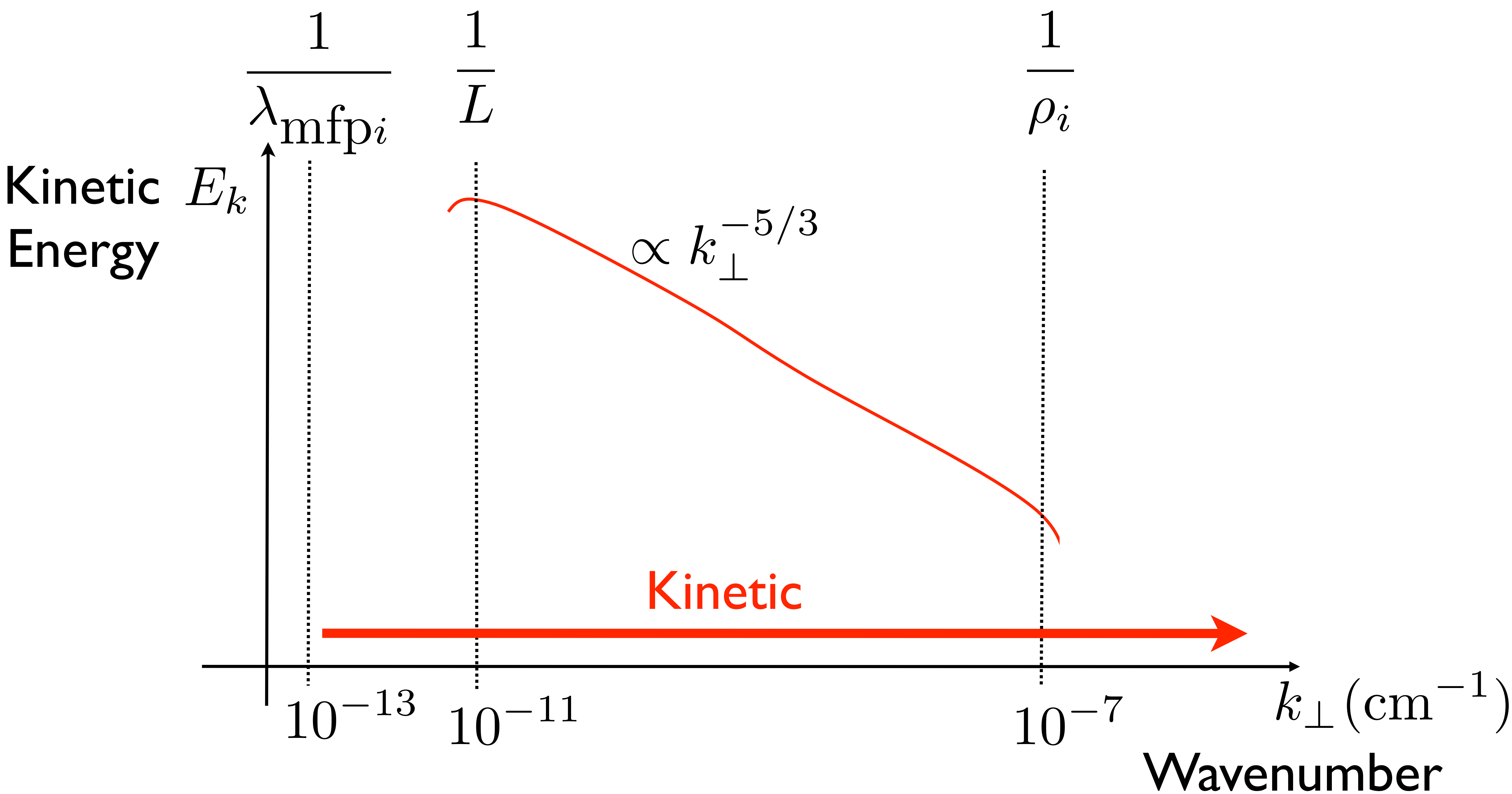
Interstellar medium



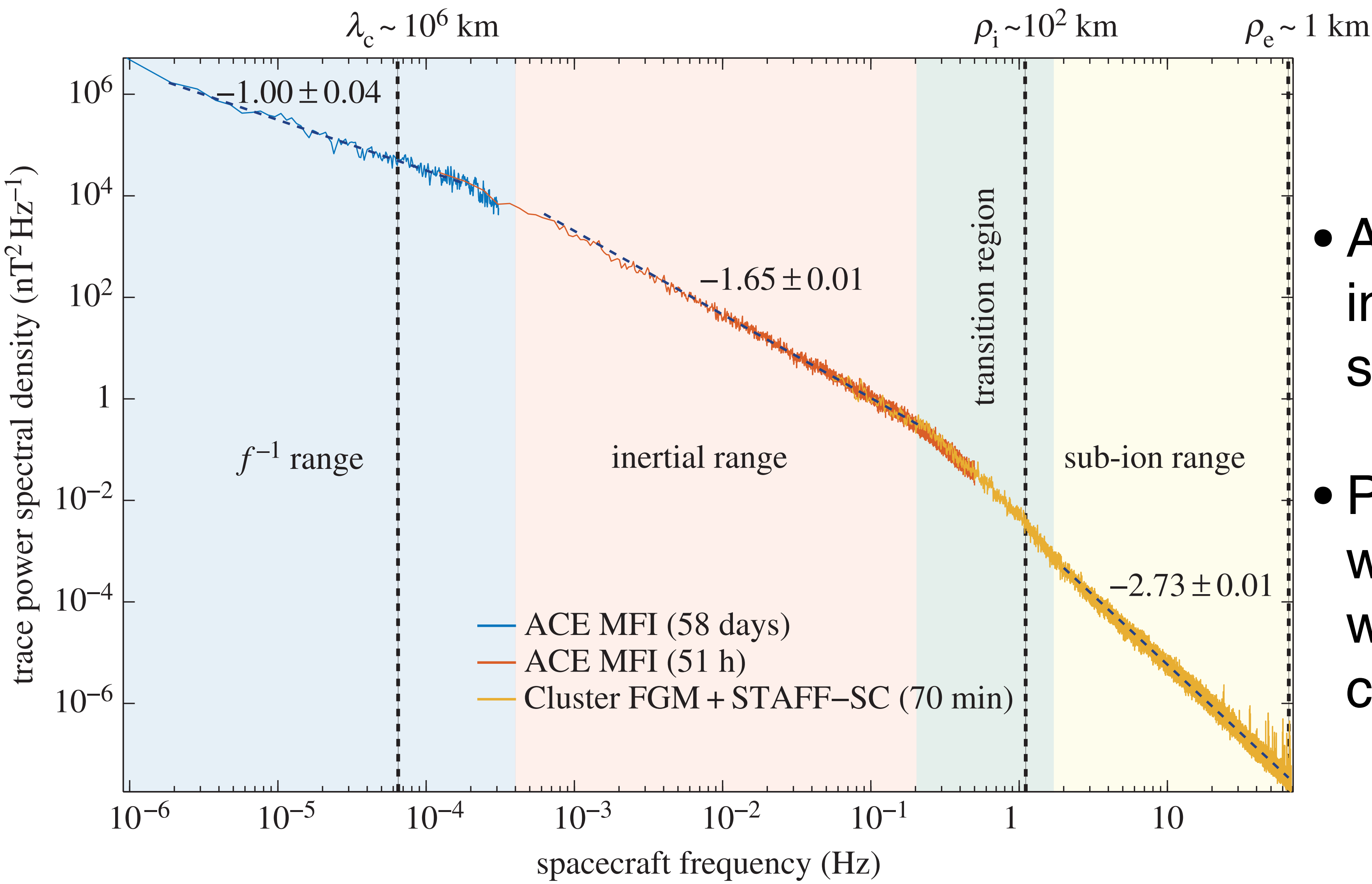
Black hole accretion disk



Solar wind energy spectrum



Solar wind energy spectrum at 1AU



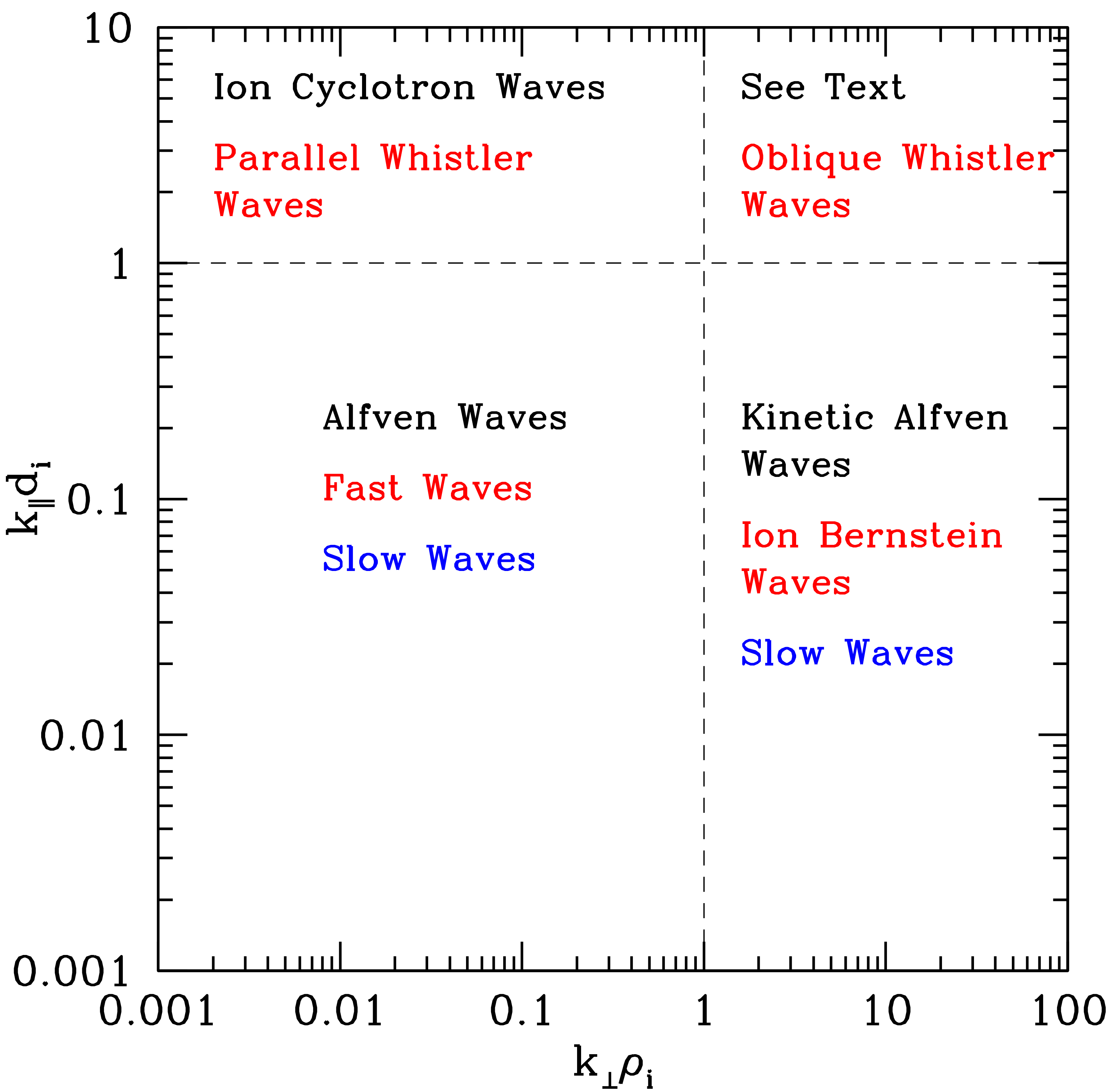
From Kiyani et al (2015).

- Alfvénic inertial range transitions into something else at the spectral break
- Proposed to be kinetic Alfvén waves, magnetosonic whistler waves, ion cyclotron waves, or current sheets

What happens when the cascade reaches kinetic scales?

Wave modes

Ion inertial length
$d_i = c/\omega_{pi}$



Ion gyroradius
$\rho_i = v_{thi}/\Omega_{ci}$

From TenBarge et al (2012)

Wave modes

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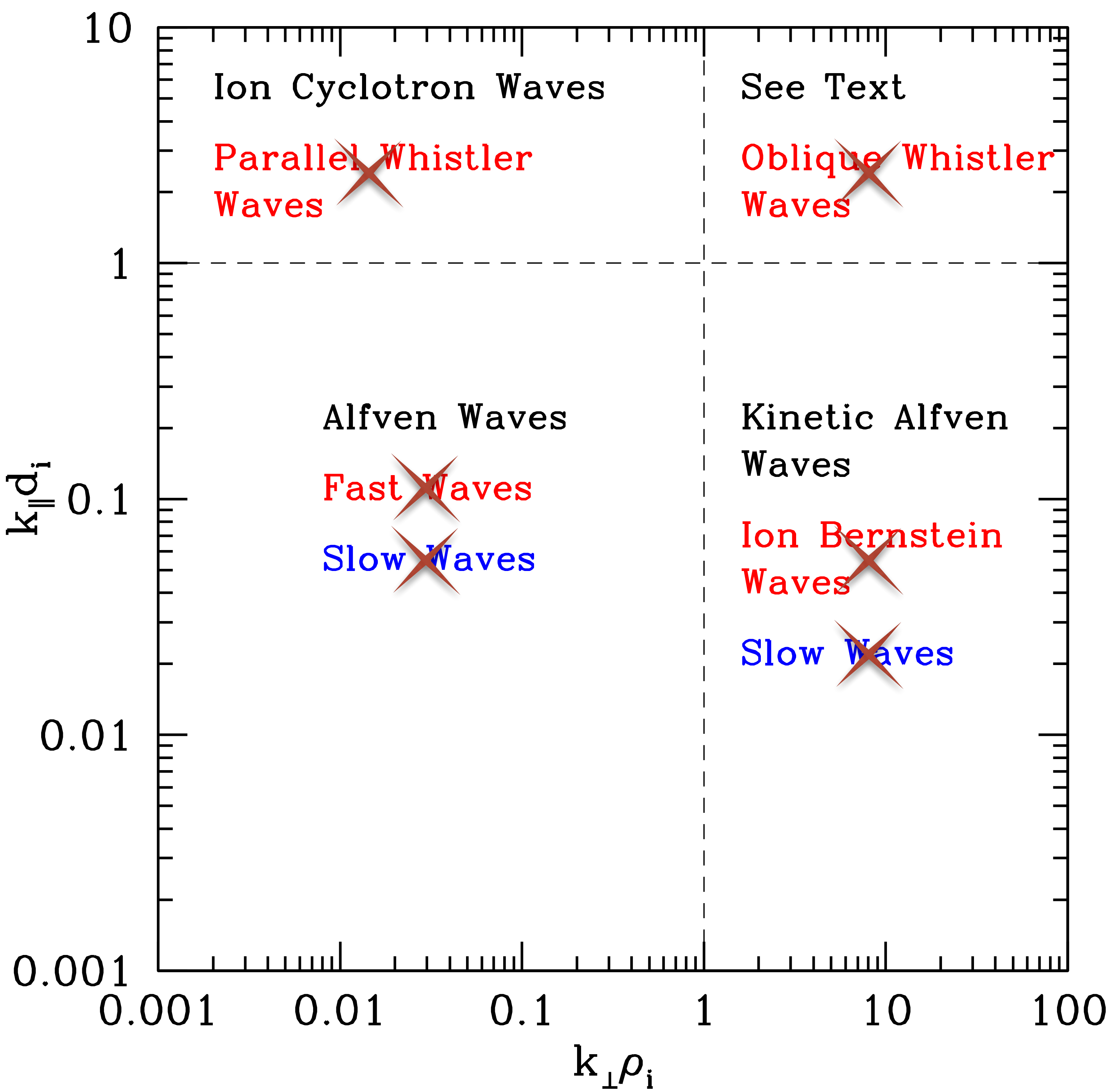

Ion gyroradius
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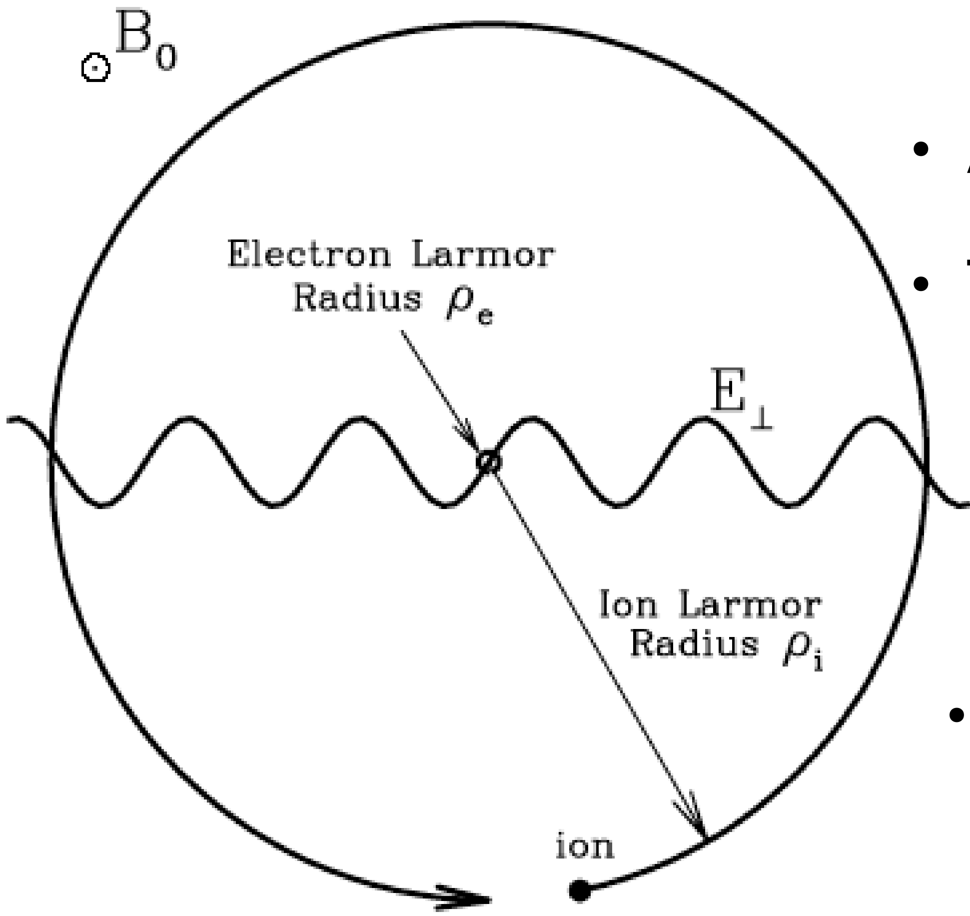
Kinetic Alfven waves



- At scales $l \ll \rho_i$, the ions decouple from the turbulence
- The turbulence is supported by electron motion alone

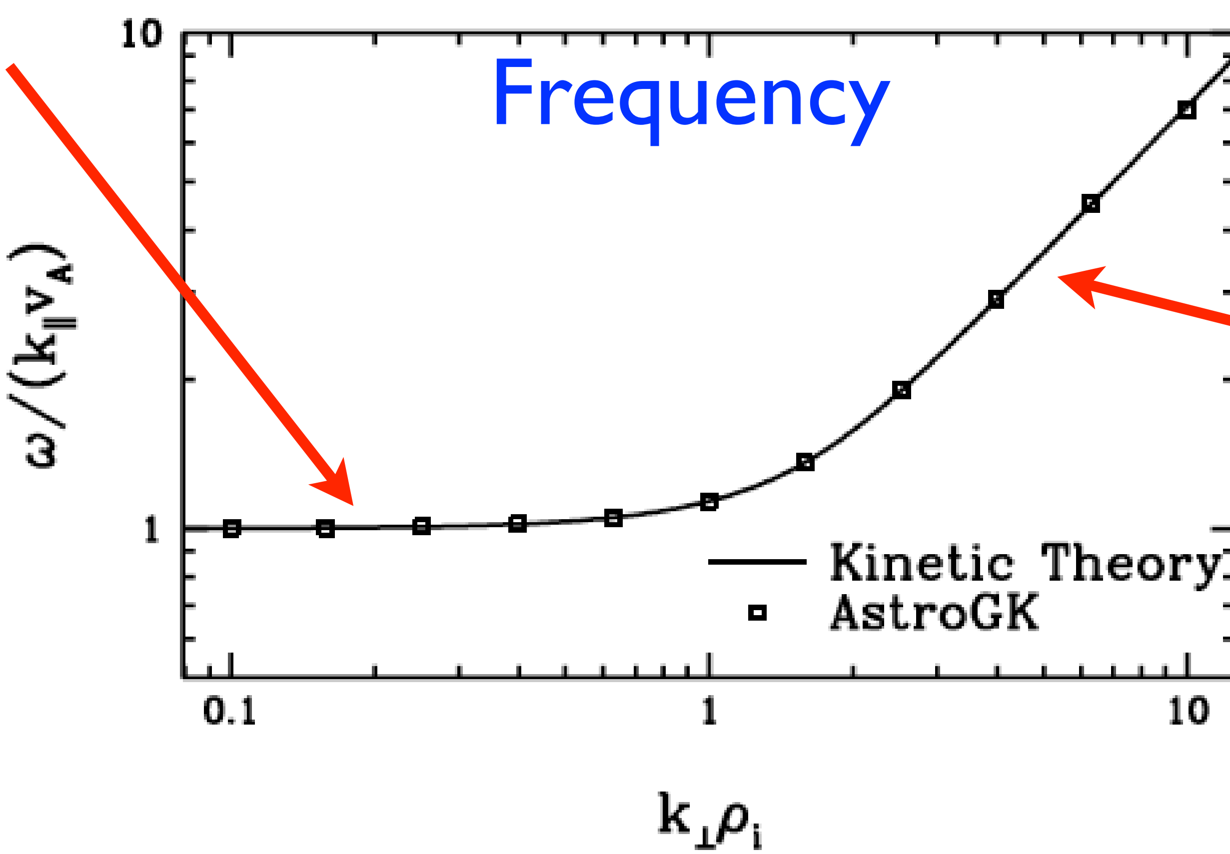
$$u_\perp = u_e$$

- Ampere's law: $\mathbf{J} = \sum_s q_s n_s \mathbf{u}_s = -en\mathbf{u}_e = \frac{4\pi}{c}\nabla \times \mathbf{B}$

$$u_\perp \propto k_\perp B_\perp$$

Alfven Waves

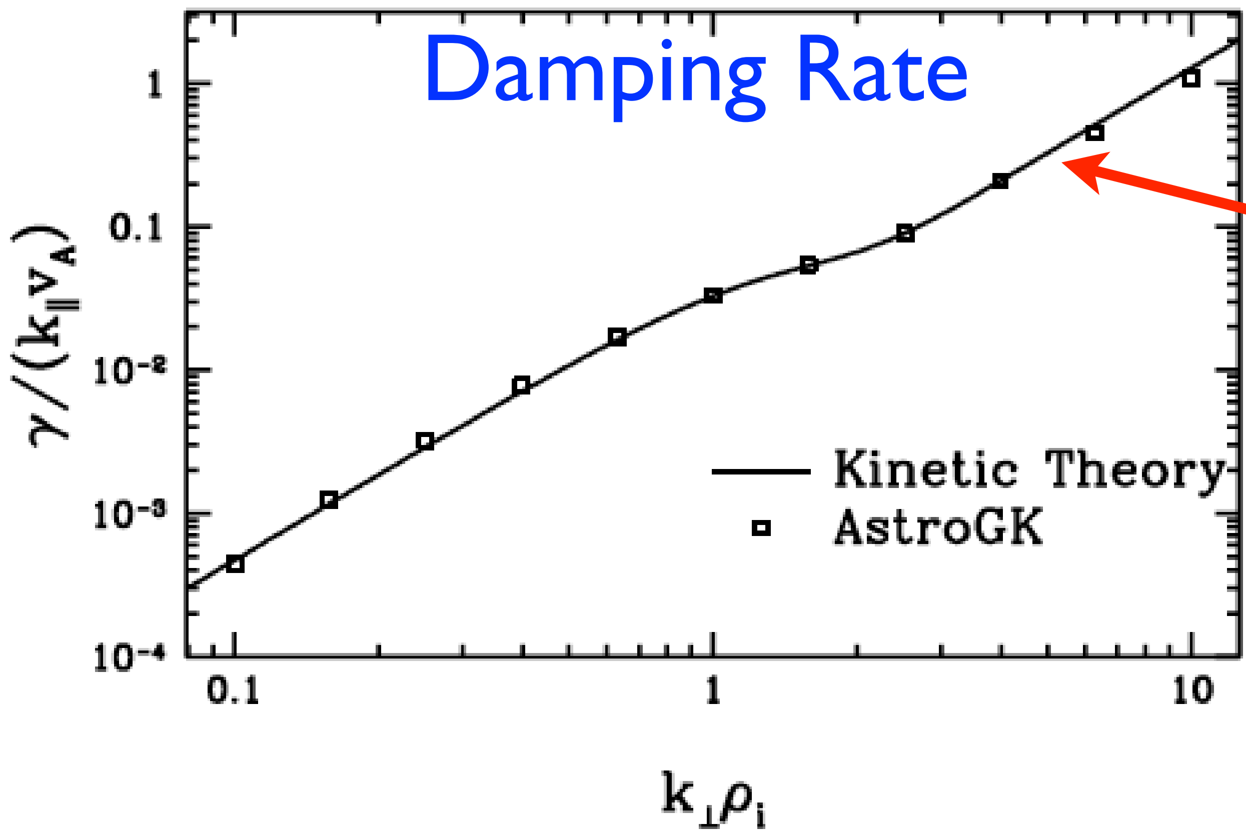
$$\omega = k_{\|} v_A$$



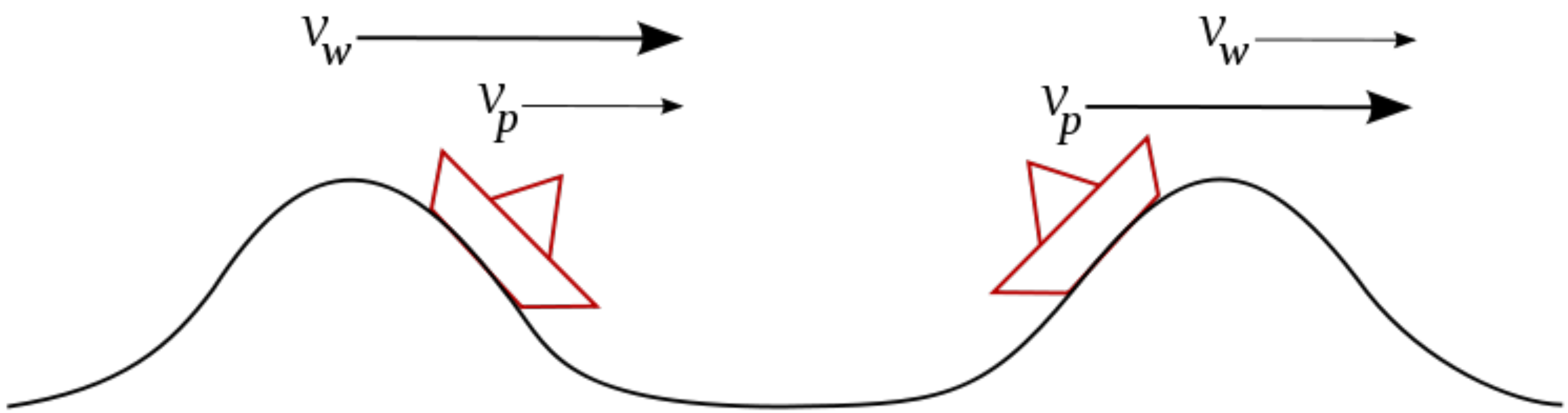
Kinetic Alfven Waves

$$\omega = k_{\|} v_A k_\perp \rho_i / \sqrt{\beta_i}$$

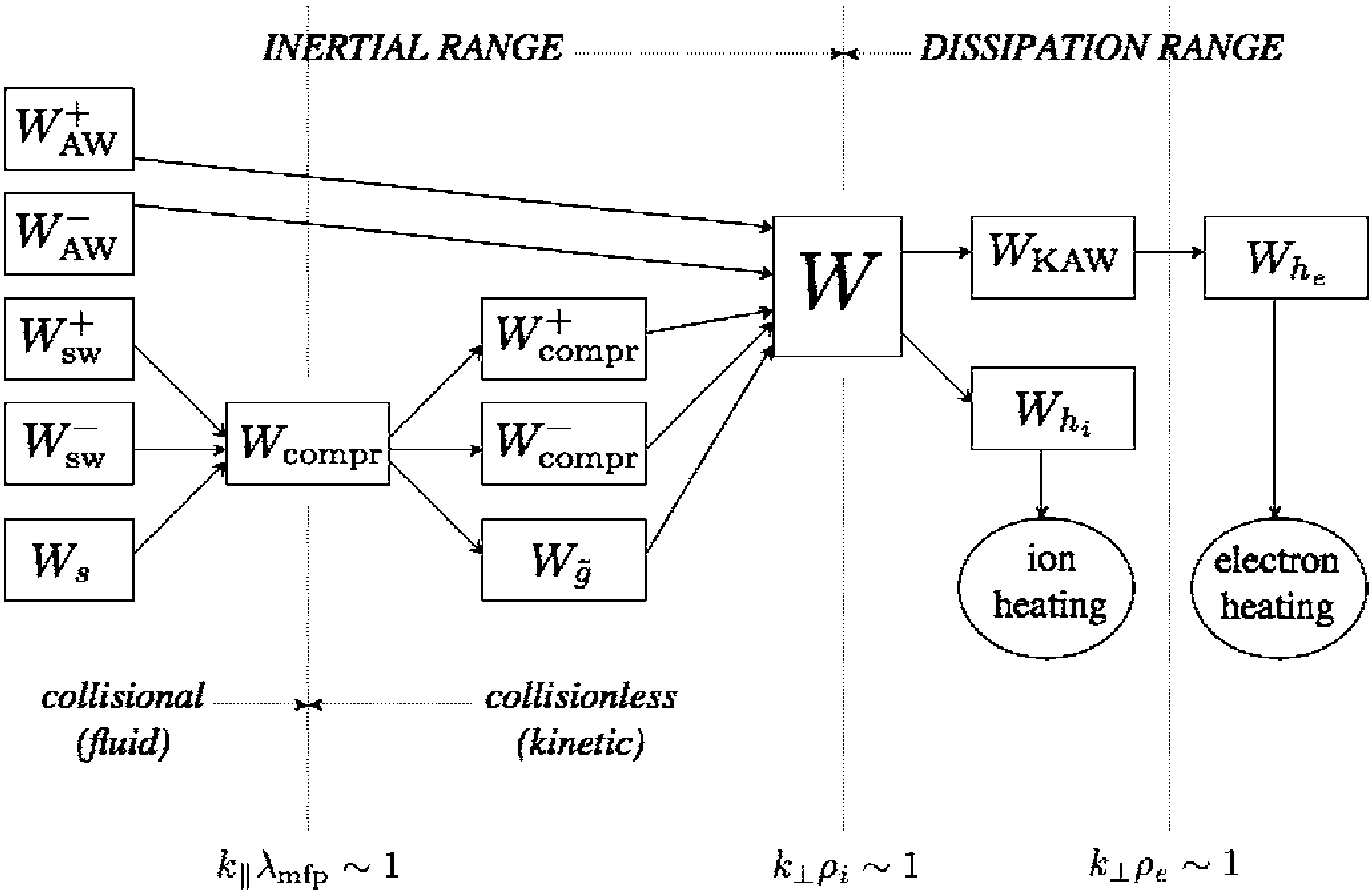
Landau damping



Kinetic Dissipation
at $l \leq \rho_i$



Multiple cascades [Schekochihin et al (2009)]



From Schekochihin et al (2009)

KAW equations [Schekochihin et al (2009)]

When $k_\perp \rho_i \gg 1$

$$\frac{\delta n_e}{n_{0e}} = -\frac{Ze\varphi}{T_{0i}} = -\frac{2}{\sqrt{\beta_i}}\frac{\Phi}{\rho_i v_A},$$

$$u_{\|e} = \frac{c}{4\pi e n_{0e}}\nabla_\perp^2 A_\| = -\frac{\rho_i \nabla_\perp^2 \Psi}{\sqrt{\beta_i}}, \qquad u_{\|i} = 0,$$

$$\frac{\delta B_\|}{B_0} = \frac{\beta_i}{2}\left(1+\frac{Z}{\tau}\right)\frac{Ze\varphi}{T_{0i}} = \sqrt{\beta_i}\left(1+\frac{Z}{\tau}\right)\frac{\Phi}{\rho_i v_A}, = -\frac{\beta_i}{2}\left(1+\frac{Z}{\tau}\right)\frac{\delta n_e}{n_{0e}}$$

$$\frac{\partial \Psi}{\partial t} = v_A\left(1+\frac{Z}{\tau}\right)\hat{\mathbf{b}}\cdot\nabla\Phi,$$

$$\frac{\partial \Phi}{\partial t} = -\frac{v_A}{2+\beta_i(1+Z/\tau)}\hat{\mathbf{b}}\cdot\nabla\left(\rho_i^2\nabla_\perp^2\Psi\right)$$

$$\hat{\mathbf{b}}\cdot\nabla = \partial/\partial z + (1/v_A)\{\Psi,\cdots\}$$

$$\{\Phi,\Psi\} = \hat{\mathbf{z}}\cdot(\nabla_\perp\Phi\times\nabla_\perp\Psi)$$

Eigenfunctions (equivalent to Elsasser fluxes)

$$\Theta_{\mathbf{k}}^{\pm} = \sqrt{\left(1+\frac{Z}{\tau}\right)\left[2+\beta_i\left(1+\frac{Z}{\tau}\right)\right]}\frac{\Phi_{\mathbf{k}}}{\rho_i} \mp k_\perp \Psi_{\mathbf{k}}$$

KAW cascade [Schekochihin et al (2009)]

$$\Psi_\lambda \sim \sqrt{1+\beta_i}\,\frac{\lambda}{\rho_i}\,\Phi_\lambda$$

$$\omega_{\mathbf{k}} = \pm\sqrt{\frac{1+Z/\tau}{2+\beta_i\,(1+Z/\tau)}}\,k_\perp\rho_i k_\parallel v_A$$

Constant energy flux

$$\frac{(\Psi_\lambda/\lambda)^2}{\tau_{\mathrm{KAW}\lambda}} \sim \frac{(1+\beta_i)(\Phi_\lambda/\rho_i)^2}{\tau_{\mathrm{KAW}\lambda}} \sim \varepsilon_{\mathrm{KAW}} = \mathrm{const}$$

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Critical balance: $\tau_{NL} \sim \tau_{KAW}$

$$\tau_{\mathrm{KAW}\lambda} \sim \frac{\lambda^2}{\Phi_\lambda} \sim \frac{1}{\sqrt{1+\beta_i}}\,\frac{\rho_i}{\lambda}\,\frac{v_A}{l_{\parallel\lambda}}$$

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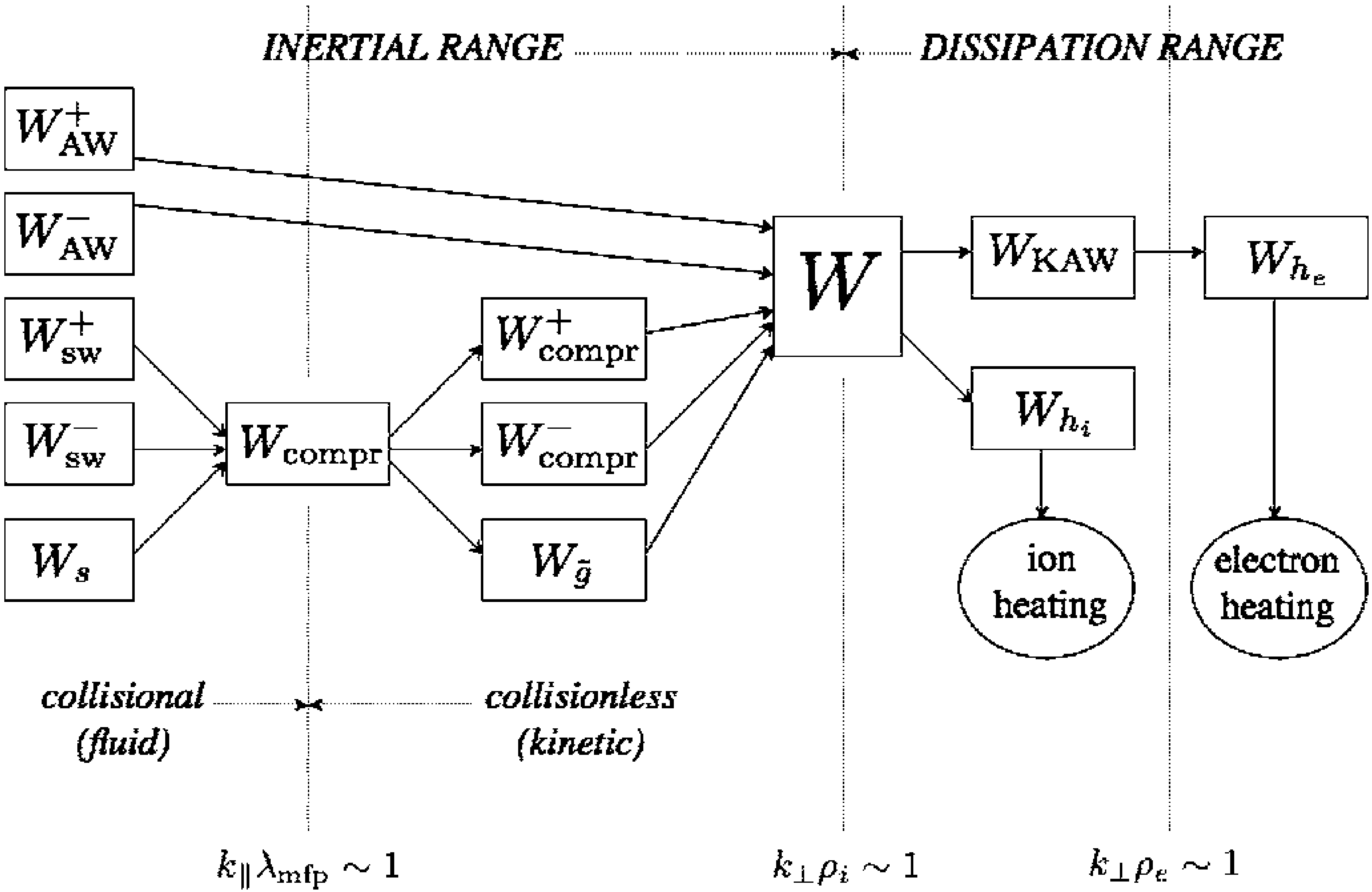
$$\Phi_\lambda \sim \left(\frac{\varepsilon_{\mathrm{KAW}}}{\varepsilon}\right)^{1/3} \frac{v_A}{(1+\beta_i)^{1/3}}\, l_0^{-1/3} \rho_i^{2/3} \lambda^{2/3} \longrightarrow E_{E_\perp} \propto k_\perp^{-1/3}, \quad E_{B_\perp} \propto k_\perp^{-7/3}$$

$$l_{\parallel\lambda} \sim \left(\frac{\varepsilon}{\varepsilon_{\mathrm{KAW}}}\right)^{1/3} \frac{l_0^{1/3} \rho_i^{1/3} \lambda^{1/3}}{(1+\beta_i)^{1/6}} \longrightarrow k_\parallel \propto k_\perp^{1/3}$$

Multiple cascades [Schekochihin et al (2009)]

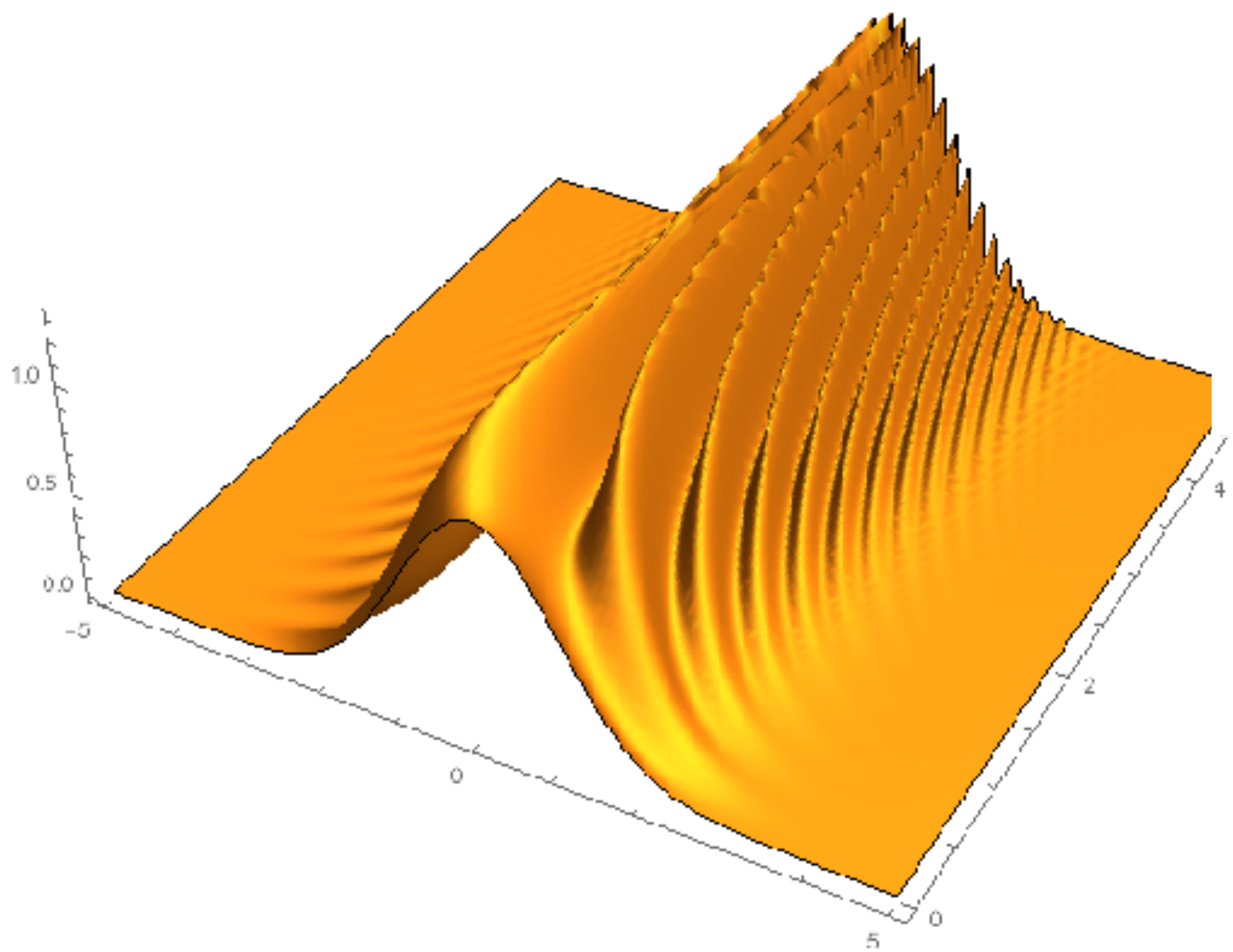


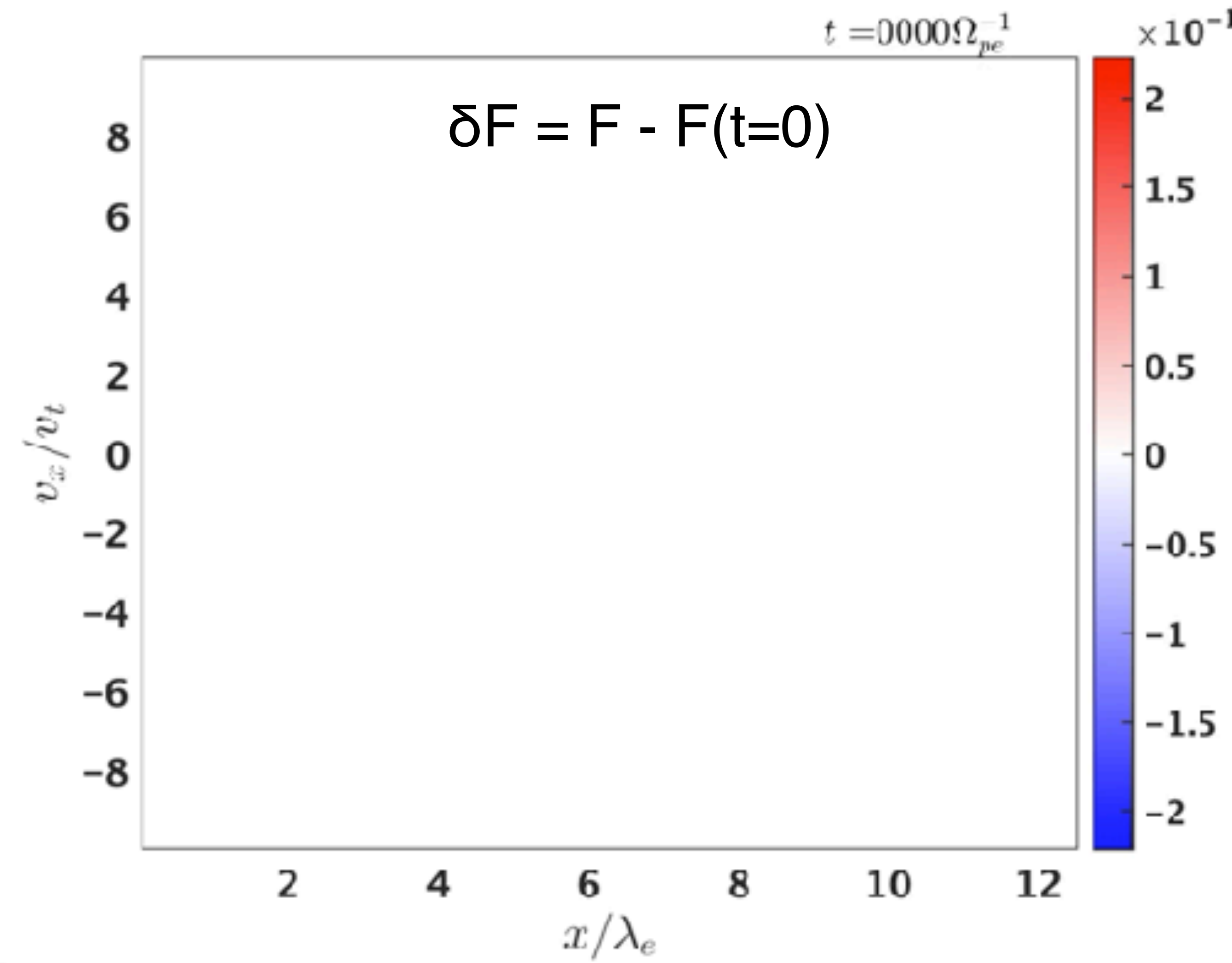
From Schekochihin et al (2009)

Linear phase mixing

$$\frac{\partial f_s}{\partial t} + v\frac{\partial f_s}{\partial x} = 0$$

$$f(x,v,t) = [1 + A\left(\sin(k_0 x)\cos(k_0 v t) - \cos(k_0 x)\sin(k_0 v t)\right)] e^{-v^2/v_t^2}$$

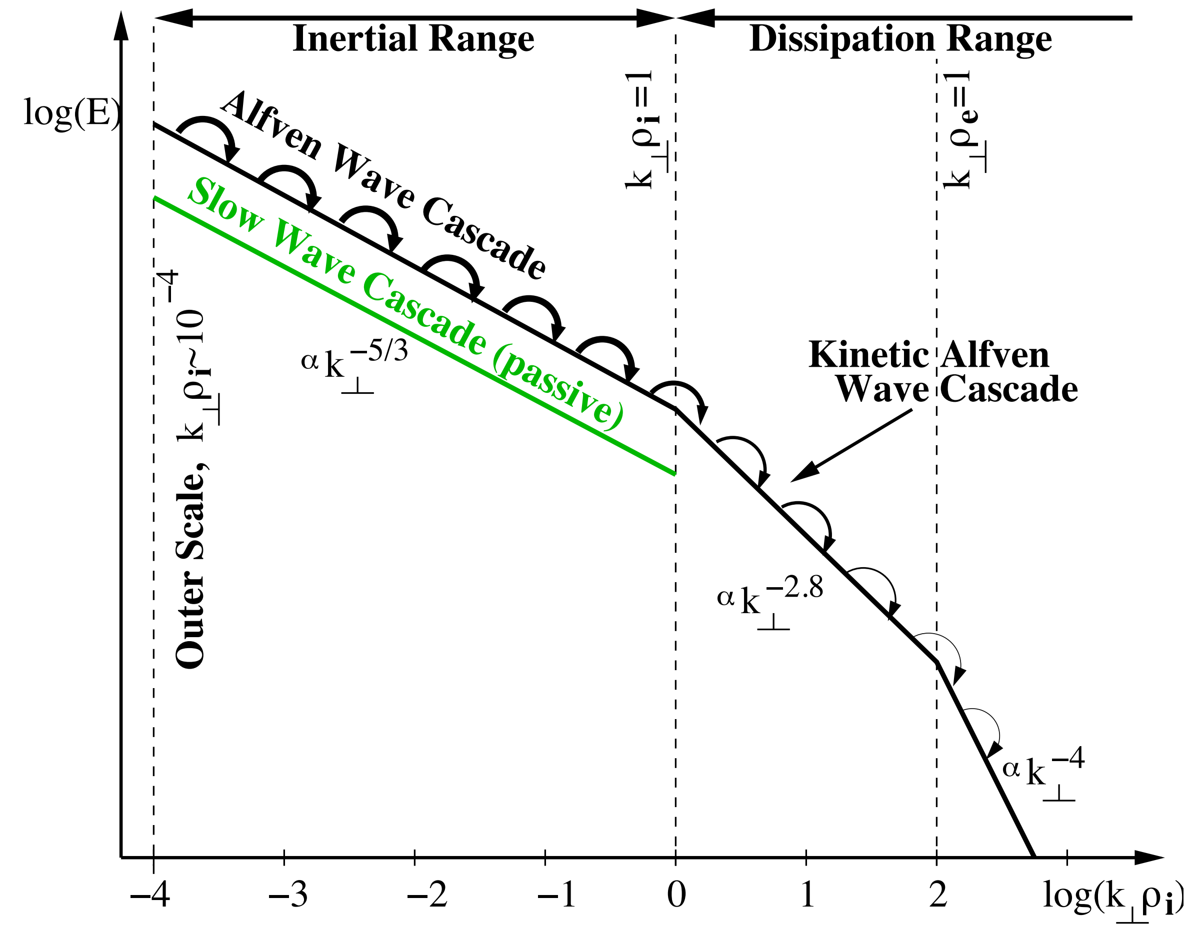


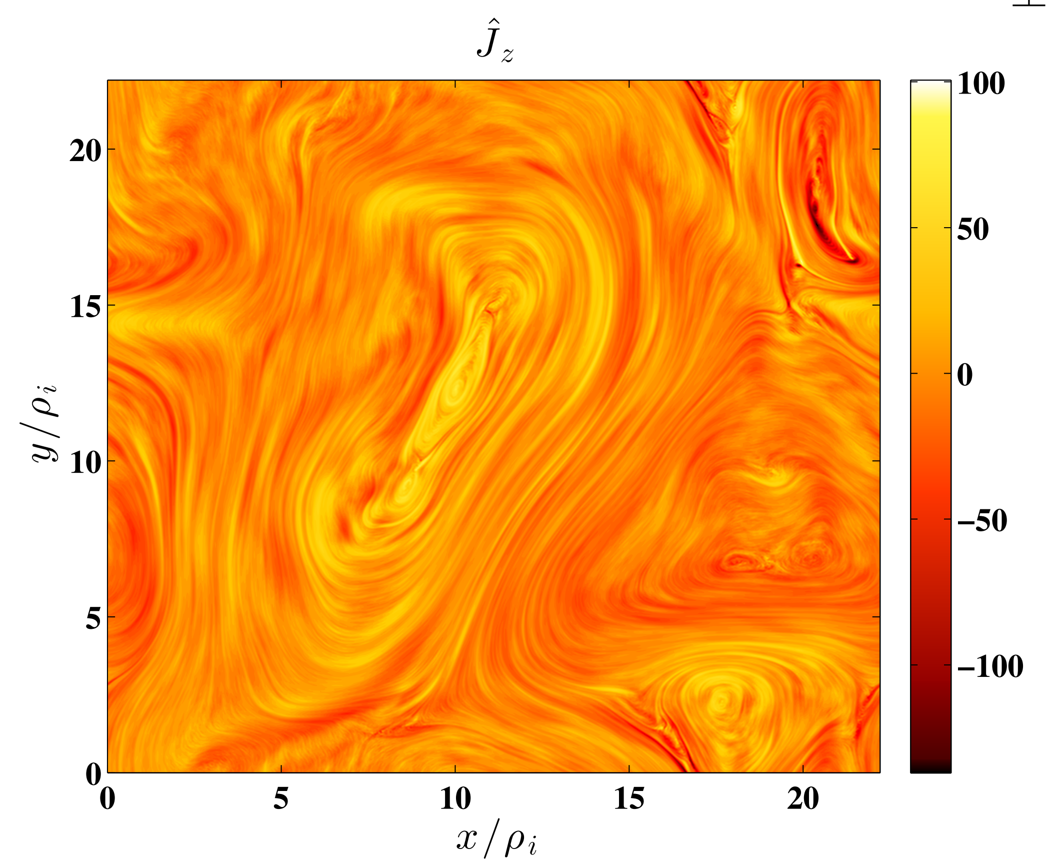


Linear phase mixing: $\dfrac{\delta v_\parallel}{v_{\mathrm{th}i}} \sim \dfrac{l_{\parallel\lambda}}{v_{\mathrm{th}i}\,\tau_{h\lambda}} \sim \dfrac{1}{\sqrt{\beta_i(1+\beta_i)}} \sim 1$

Turbulence at kinetic scales

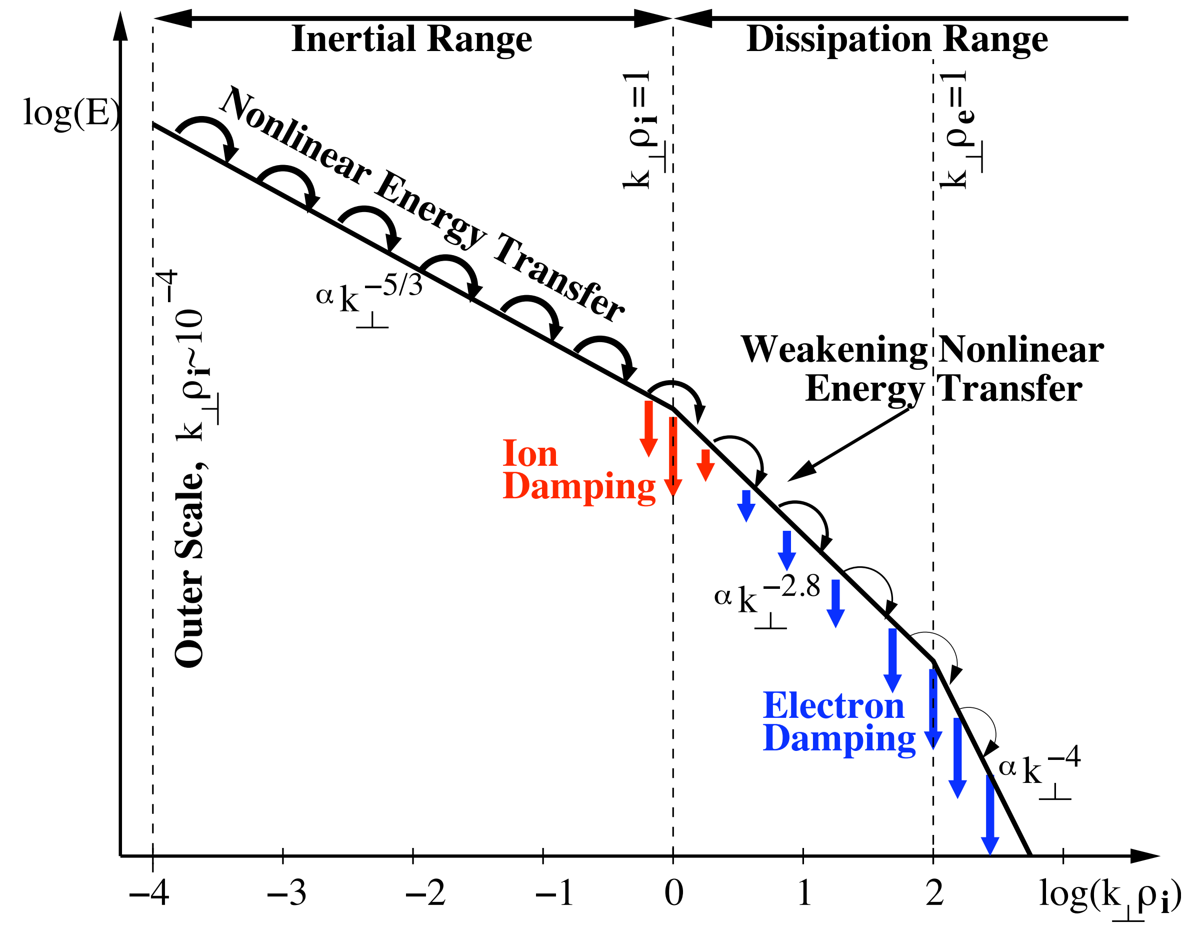
- Anisotropic cascade of MHD Alfvén waves transitions to a cascade of kinetic Alfvén waves at the ion Larmor radius.

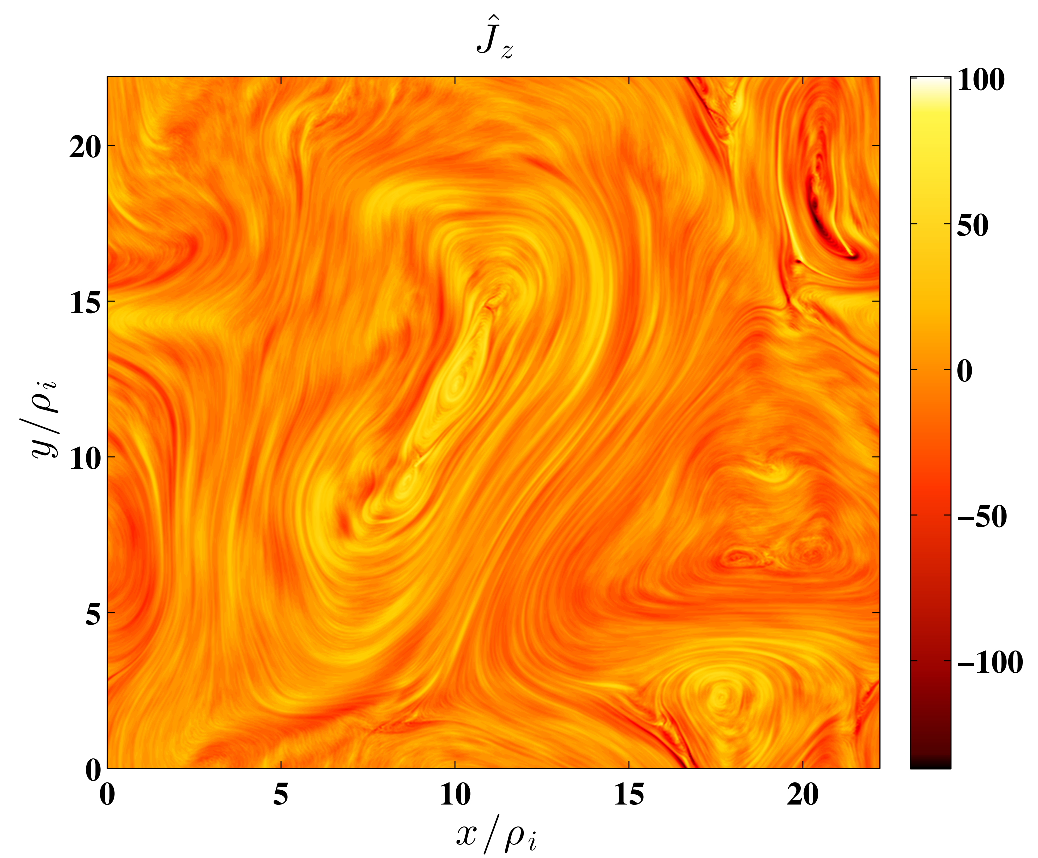




Turbulence at kinetic scales

- Anisotropic cascade of MHD Alfvén waves transitions to a cascade of kinetic Alfvén waves at the ion Larmor radius.
- Dissipation begins at ion kinetic scales in the form wave-particle interactions (Landau, transit-time, cyclotron, …).





Turbulence at kinetic scales

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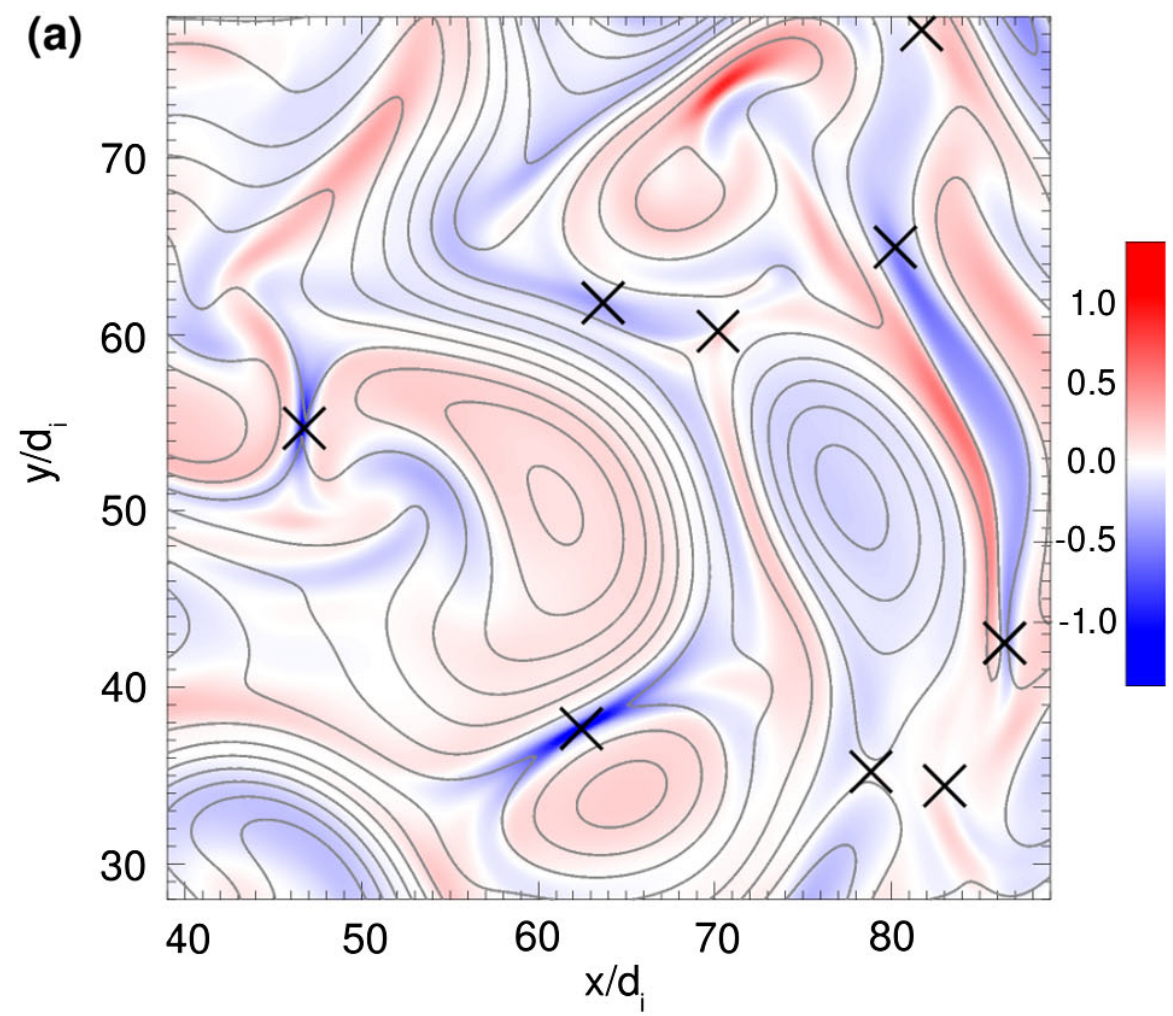
- Dissipation begins at ion kinetic scales in the form wave-particle interactions (Landau, transit-time, cyclotron, …).

- Current sheets also form at ion scales and may be responsible for dissipation.



Shaded contours of j_z together with A_z isolines, and its X points (black crosses) [Servidio et al PRL (2012)].

Turbulence at kinetic scales

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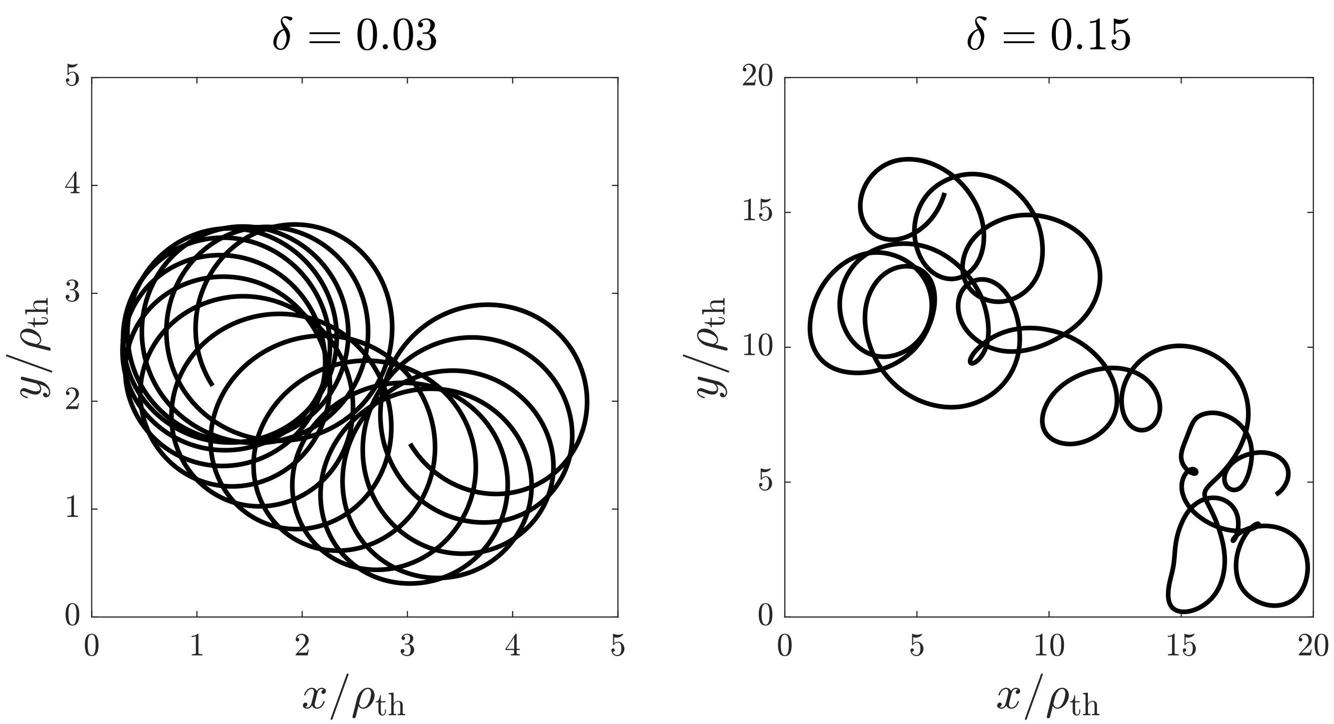
- Dissipation begins at ion kinetic scales in the form wave-particle interactions (Landau, transit-time, cyclotron, …).
- Current sheets also form at ion scales and may be responsible for dissipation.
- Energy can also dissipate via other mechanisms, such as stochastic heating.



Trajectories of test-particle protons interacting with a spectrum of randomly phased AWs and KAWs for different values of the stochasticity parameter δ [Hoppock et al JPP (2019)].

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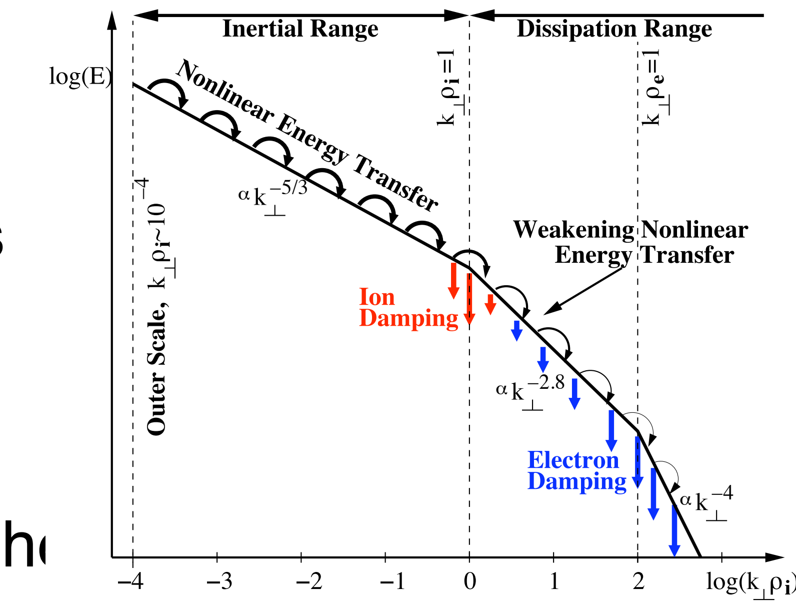
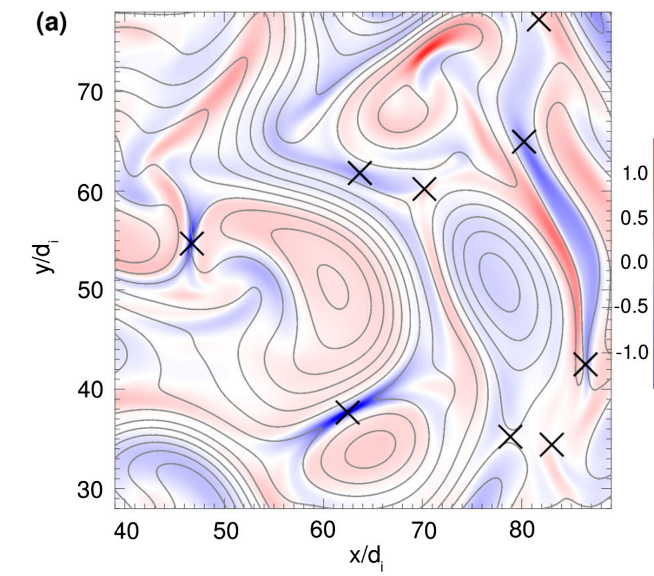
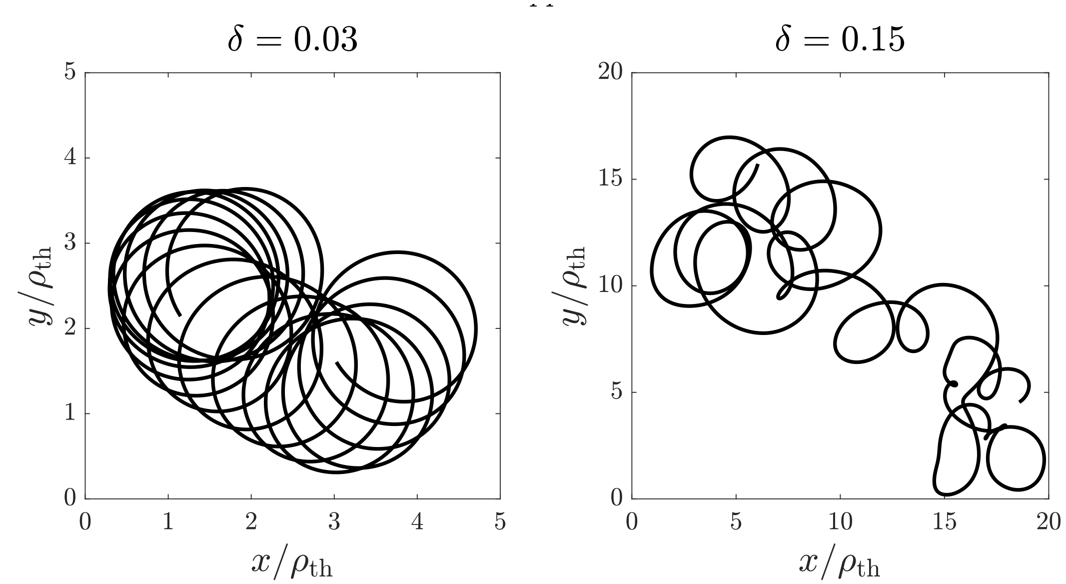
- Current sheets also form at ion scales and may be responsible for dissipation.
- Energy can also dissipate via other mechanisms, such as stochastic heating.
- Which mechanism is dominant in weakly collisional kinetic plasmas, and how do they each heat the plasma?

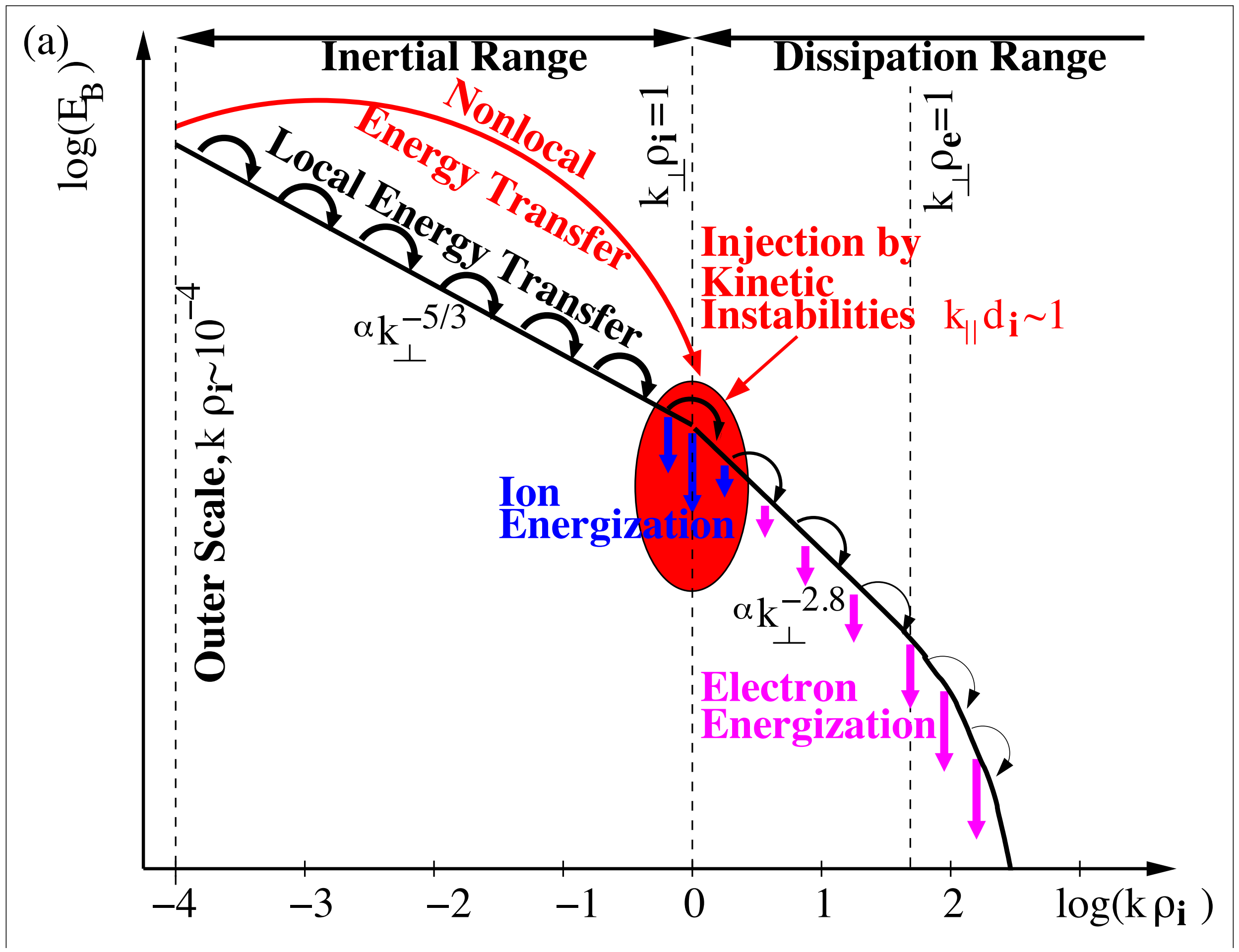


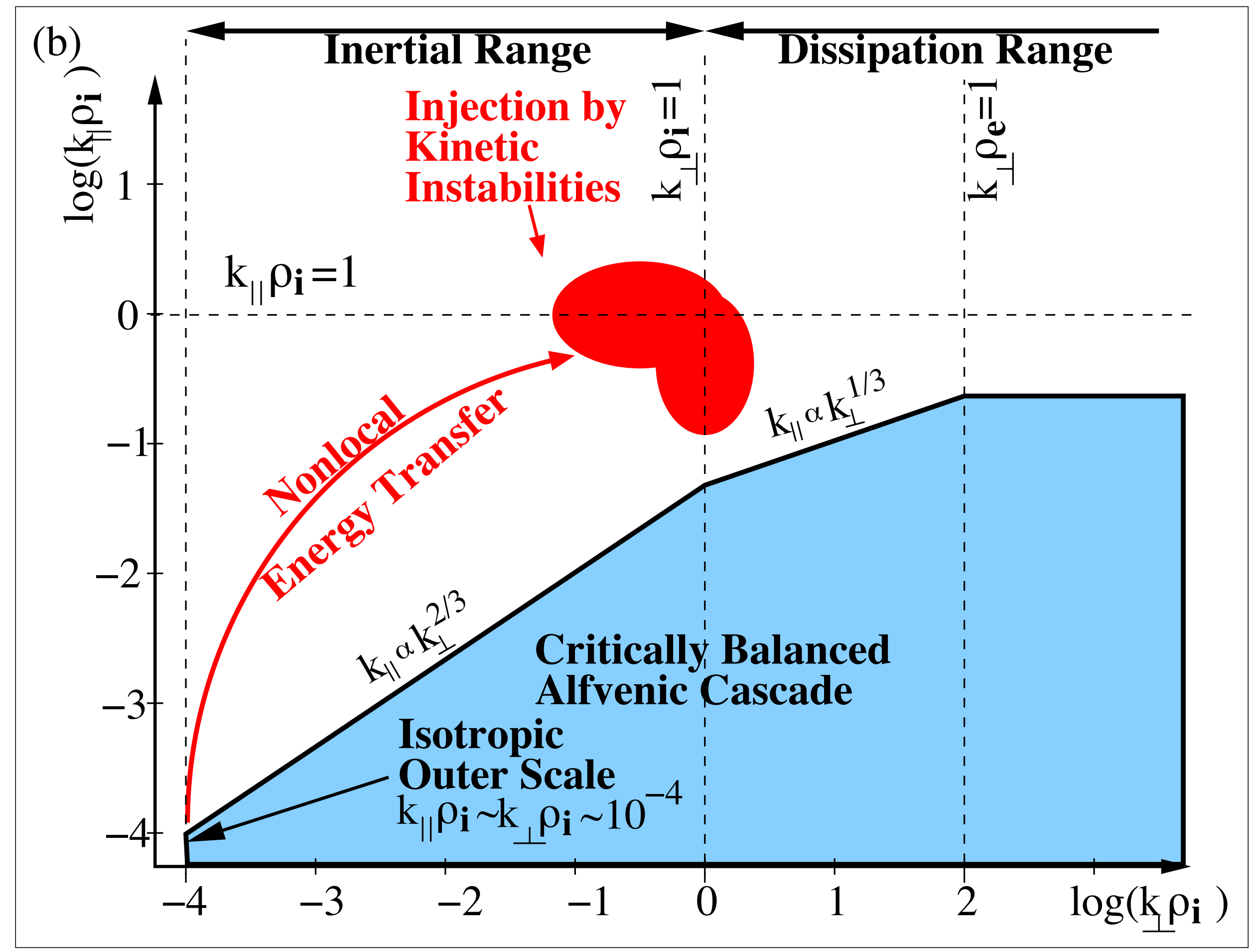




What about instabilities?

Role of instabilities





Howes Phil Trans A (2015)

Supplemental material

Entropy cascade [Schekochihin et al (2009)]

Linear phase mixing: $\frac{\delta v_\parallel}{v_{\text{th}i}} \sim \frac{l_{\parallel\lambda}}{v_{\text{th}i}\tau_{h\lambda}} \sim \frac{1}{\sqrt{\beta_i(1+\beta_i)}} \sim 1$

Nonlinear phase mixing

$$\frac{\delta v_\perp}{v_{\text{th}i}} = \frac{|v_\perp - v'_\perp|}{v_{\text{th}i}} \sim \frac{1}{k_\perp \rho_i}$$

$$\tau_{h\lambda} \sim \frac{\rho_i}{\lambda}\tau_{\text{KAW}\lambda} \sim \left(\frac{\varepsilon}{\varepsilon_{\text{KAW}}}\right)^{1/3}(1+\beta_i)^{1/3}\frac{l_0^{1/3}\rho_i^{1/3}\lambda^{1/3}}{v_A}$$

$$h_{i\lambda} \sim \frac{n_{0i}}{v_{\text{th}i}^3}\left(\frac{\varepsilon_h}{\varepsilon}\right)^{1/2}\left(\frac{\varepsilon}{\varepsilon_{\text{KAW}}}\right)^{1/6}\frac{(1+\beta_i)^{1/6}}{\sqrt{\beta_i}}\, l_0^{-1/3}\rho_i^{1/6}\lambda^{1/6}$$

$$\frac{\delta v_\perp}{v_{\text{th}i}} \sim \frac{1}{k_\perp \rho_i} \sim (\nu_{ii}\tau_{\rho_i})^{3/5} = \text{Do}^{-3/5}$$